BOOK REVIEWS

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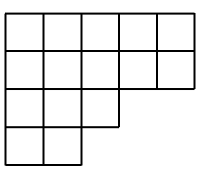
Young tableaux: With applications to representation theory and geometry, by William Fulton, Cambridge Univ. Press, New York, 1997, ix + 260 pp., ISBN 0-521-56144-2 (hardcover), ISBN 0-521-56724-6 (paperback)

The extensive theory of the combinatorics of tableaux provides tools for giving very precise information about

(i) the representations of symmetric groups, general linear groups, special linear groups, unitary groups, . . . , and
(ii) the geometry of Grassmannians, flag varieties, and Schubert varieties.

This book is one of the very few which gives an introduction to this theory. Many parts of theory which were previously available only in a scattered form in the specialized literature are presented here in a coherent fashion.

The main players in the theory are partitions, tableaux and standard tableaux. A *partition* of n is a collection of n boxes in a corner (gravity goes up and to the left). A *tableau* of shape λ is a filling of the boxes of λ with positive integers such that the rows are weakly increasing and the columns are strictly increasing. A *standard tableau* of shape λ is a tableau of shape λ such that each of the numbers $1, 2, \dots, n$ occurs exactly once.



Partition

1	2	3	8	14
4	6	9	11	15
5	7	13		
10	12			

Standard tableau

1	1	2	2	3
2	3	3	5	6
5	6	6		
6	7			

Tableau

The strong relationship between tableaux and representation theory is reflected in the following facts:

(1) The irreducible representations S^λ of the symmetric S_n are indexed by partitions with n boxes,
(2) $\dim(S^\lambda)$ is the number of standard tableaux of shape λ,
(3) There is a unique partition with $\leq m$ rows corresponding to each irreducible (holomorphic) representation of $GL_m(\mathbb{C})$,

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(4) If λ is the partition corresponding to an irreducible representation M of $GL_m(\mathbb{C})$, then $\dim(M)$ is the number of tableaux of shape λ with entries from the set $\{1, 2, \dots, m\}$.

One of the main theorems of representation theory, the Borel-Weil-Bott theorem, says that the irreducible representations of $GL_m(\mathbb{C})$ can be constructed from the geometry of flag varieties: the irreducible $GL_m(\mathbb{C})$-modules are the spaces $H^0(G/P, L^\lambda)$ of global sections of (appropriate) line bundles L^λ on the flag varieties G/P. This indicates how (and why) there is a connection between tableaux theory and the geometry of flag varieties (Grassmannians are special cases of general flag varieties).

A summary of the contents of this book will provide a good survey of the field and a description of many of the main results. As with all subjects, there are many possible points of view, choices of material, and ways of presenting each topic; here the effort has been to be brief and faithful to the presentation of the book under review, and opinions are given following the summary.

Following the book, it is natural to divide the material into three (interrelated) parts.

Part I: The calculus of tableaux

Row insertion is a specific combinatorial way of adding a "letter" $i \in \{1, 2, \dots, n\}$ to a tableau T to get a new tableau $(T \leftarrow i)$ which has one more box than before. By iterating this procedure one can row insert any "word" $w = i_1 i_2 \cdots i_r$ into the Young tableau T to get a new tableau $T \leftarrow w$ with r new boxes.

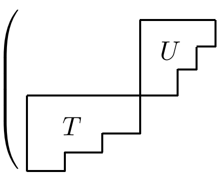
Jeu de taquin is a combinatorial game for sliding out a "hole" (unfilled box) in a filling that is a tableau except for the unfilled box. By iterating the game, one can remove several holes, and one of the magical things is that it does not matter in which order the holes are removed. The *rectification* $\mathrm{Rect}(T)$ of a filling is the Young tableau obtained by sliding out all the holes of a partially filled tableau T.

The *plactic algebra* $R_{[n]}$ is the free associative algebra generated by the symbols $1, 2, \dots, n$ modulo the *Knuth equivalence relations*

$$yzx = yxz \quad \text{if} \quad x < y \le z, \qquad \text{and} \qquad xzy = zxy \quad \text{if} \quad x \le y < z.$$

The generators are ordered in the usual order $1 < 2 < \cdots < n$. The word $w(T)$ of a column strict tableau T is obtained by reading the entries of T from left to right and bottom to top. Surprisingly, one can actually find a basis of this quotient ring: the set $\{w(T) \mid T \text{ column strict tableaux}\}$ is a basis of the plactic algebra. If $v = i_1 i_2 \cdots i_r$ is a word, then $v = w(\emptyset \leftarrow v)$ in the plactic algebra, where $w(\emptyset \leftarrow v)$ is the word of the tableau obtained by row inserting v into the empty tableau $\emptyset$.

One may define products of tableaux by jeu de taquin

$T \cdot U = \mathrm{Rect}\Bigg($ 

$\Bigg)$, or by row insertion, $\quad T \star U = (T \longleftarrow w(U)).$

Amazingly,

$$T \cdot U = T \star U \quad \text{and} \quad w(T \cdot U) = w(T)w(U) \text{ in the plactic algebra.}$$

The capstone of the proofs of these facts is to show that the tableau $\emptyset \leftarrow v$ of a word v is completely determined by the increasing subsequences of v.

The Schur function is the polynomial $s_\lambda \in \mathbb{Z}[x_1, \dots, x_n]$ defined by

$$s_\lambda(x) = \sum_T x^T, \qquad \text{where} \quad x^T = x_1^{(\#\text{ of 1's in } T)} \cdots x_n^{(\#\text{ of } n\text{'s in } T)}$$

and the sum is over all tableaux T of shape λ. The Schur function is a symmetric polynomial, i.e. $s_\lambda(x) \in \mathbb{Z}[x_1, \dots, x_n]^{S_n}$. There is an injective algebra homomorphism

$$\begin{array}{ccc} \mathbb{Z}[x_1, \dots, x_n]^{S_n} & \longrightarrow & R_{[n]} \\ s_\lambda(x) & \longmapsto & \sum_T w(T) \end{array}$$

which means that computations with Schur functions can be done in the plactic algebra with the use of jeu de taquin and row insertion.

Many identities in the theory of symmetric functions are amenable to proofs by row insertion methods. In particular, row insertion can be used to give bijective proofs of the following identities:

$$n! = \sum_\lambda (f^\lambda)^2, \quad (x_1 + \cdots + x_n)^n = \sum_\lambda s_\lambda(x) f^\lambda, \quad \prod_{i,j} \frac{1}{1 - x_i y_j} = \sum_\lambda s_\lambda(x) s_\lambda(y),$$

where f^λ is the number of standard tableaux of shape λ. The resulting bijections (and their immediate relatives) are the *Robinson-Schensted-Knuth correspondences.*

If the boxes of a partition λ are a subset of the boxes of ν, then ν/λ denotes the boxes of ν which are not in λ. The *Littlewood-Richardson rule* says that

$$s_\mu(x) s_\lambda(x) = \sum_\lambda c^\nu_{\mu\lambda} s_\nu(x),$$

where $c^\nu_{\mu\lambda}$ is the number of *Littlewood-Richardson fillings* of ν/λ which have content μ. The classical Pieri formulas in the theory of the Grassmannian and the Clebsch-Gordan formula from the theory of the unitary group are special cases of this formula. The proof of the Littlewood-Richardson rule is a beautiful combination of the tools developed thus far: row insertion, jeu de taquin, and the plactic algebra. It proceeds by showing that if $U_\circ$ is any fixed tableau of shape μ and $V_\circ$ is any fixed tableau of shape ν, then

$$\{\text{column strict fillings } S \text{ of } \nu/\lambda \text{ such that } \mathrm{Rect}(S) = U_\circ\} \xleftrightarrow{1-1}$$
$$\{[T, U] \mid T \text{ is a tableau of shape } \lambda,\ U \text{ is a tableau of shape } \mu \text{ and } T \cdot U = V_\circ\},$$

and

$$\{\text{Littlewood-Richardson fillings of } \nu/\lambda \text{ of content } \mu\} =$$
$$\{\text{column strict fillings } S \text{ of } \nu/\lambda \text{ such that } \mathrm{Rect}(S) = U(\mu)\},$$

where $U(\mu)$ is the tableau of shape μ obtained by filling the first row with 1's, the second row with 2's, ...

Part II: Representation Theory

The symmetric group

Let λ be a partition of n. A *tabloid* of shape λ is an equivalence class of numberings of the boxes of λ (from 1 to n) where two numberings are equivalent if corresponding rows contain the same entries. Let M^λ be the span of all tabloids $\{T\}$ of shape λ with an action of S_n defined by

$$\sigma\{T\} = \{\sigma T\}$$

(σ permutes the numbers of T). For each numbering T of λ let

$$C(T) = \left\{ \sigma \in S_n \;\middle|\; \begin{matrix} \sigma \text{ permutes the entries of each} \\ \text{column of } T \text{ among themselves} \end{matrix} \right\} \quad \text{and}$$

$$v_T = \sum_{q \in C(T)} \operatorname{sgn}(q)\{q \cdot T\}.$$

The *Specht module* S^λ is the submodule of M^λ spanned by the elements v_T. The S^λ form a complete set of irreducible S_n-modules, and the set

$$\{v_T \mid \; T \text{ is a standard tableau of shape } \lambda\} \qquad \text{is a basis of } S^\lambda.$$

There is a dual construction of the Specht modules. Let $\tilde{M}^\lambda$ be the span of all column tabloids $[T]$ of shape λ (two numberings are equivalent if corresponding *columns* contain the same entries) with an action of S_n defined by

$$\sigma[T] = \operatorname{sgn}(\sigma)[T].$$

There is a canonical surjection

$$\begin{array}{rccc} \alpha\colon & \tilde{M}^\lambda & \longrightarrow & S^\lambda \\ & [T] & \longmapsto & v_T \end{array}$$

with kernel Q^λ generated by the “quadratic relations”. The kernel Q^λ is also generated by the (very similar) “Garnir relations”.

Let R_n be the Grothendieck group of representations of S_n. Define a product on

$$R = \bigoplus_{n=0}^{\infty} R_n \quad \text{ by } \quad [V] \circ [W] = [\operatorname{Ind}_{S_k \times S_\ell}^{S_{k+\ell}}(V \otimes W)],$$

for $[V] \in R_k, [W] \in R_\ell$, an inner product by $\langle [S^\lambda], [S^\mu] \rangle = \delta_{\lambda\mu}$, and an involution by $\omega([V]) = [V \otimes S^{(1^n)}]$, for $[V] \in R_n$. ($S^{(1^n)}$ is the sign representation of S_n.) Then the map

$$\begin{array}{ccc} R & \longrightarrow & \text{symmetric polynomials} \\ [S^\lambda] & \longmapsto & s_\lambda(x) \end{array}$$

is an isometric, involution preserving, ring isomorphism. Thus, computations with representations of the symmetric group can be done with symmetric functions!

The general linear group

Let E be the standard $GL_n(\mathbb{C})$-module of dimension n. Let λ be a partition with $\leq m$ rows and let $d_1, d_2, \ldots, d_\ell$ be the lengths of the columns of λ. The *Schur module* is the quotient

$$E^\lambda = \frac{(\bigwedge^{d_1} E) \otimes \cdots \otimes (\bigwedge^{d_\ell} E)}{Q^\lambda(E)},$$

where $Q^\lambda(E)$ is the ideal of "quadratic relations". Incredibly, the character of the $GL_m(\mathbb{C})$-module E^λ is the Schur function!

$$\mathrm{Char}(E^\lambda) = s_\lambda(x).$$

The E^λ are realizations of the irreducible polynomial representations of $GL_m(\mathbb{C})$. If $D\colon GL_n(\mathbb{C}) \to \mathbb{C}$ denotes the one dimensional representation of $GL_n(\mathbb{C})$ given by $D(g) = \det(g)$, then each irreducible rational (holomorphic) representation of $GL_n(\mathbb{C})$ is of the form $E^\lambda \otimes D^{\otimes r}$ for unique λ and $r \in \mathbb{Z}$.

The symmetric group S_n acts on $E^{\otimes n} = E \otimes \cdots \otimes E$ by permuting the tensor factors, and the functor from S_n-modules to $GL_m(\mathbb{C})$-modules defined by

$$E(M) = E^{\otimes n} \otimes_{\mathbb{C}S_n} M$$

sets up a tight relationship between the representation theory of $GL_m(\mathbb{C})$ and S_n. In particular, $E(S^\lambda) = E^\lambda$.

Suppose that the conjugate of λ is the partition $(d_1^{a_1} \cdots d_s^{a_s})$. Let P be the subgroup of $G = GL_m(\mathbb{C})$ of block lower triangular matrices with block sizes d_s, $d_{s-1} - d_s, \ldots, d_1 - d_2$, and let $\chi_\lambda\colon P \to \mathbb{C}^*$ be the one dimensional representation of P given by $\chi_\lambda(p) = \det(A_1)^{a_1} \det(A_2)^{a_2} \cdots \det(A_s)^{a_s}$ where A_i is the upper left $d_i \times d_i$ submatrix of P. The Borel-Weil-Bott theorem says that, as $G = GL_m(\mathbb{C})$-modules,

$$E^\lambda \cong \Gamma(G/P, L^\lambda),$$

where $\Gamma(G/P, L^\lambda)$ is the space of global sections of the line bundle $L^\lambda = G \times_P \chi_\lambda$ over G/P. This means that there is an intimate connection between the geometry of the coset spaces G/P and the representations of $GL_m(\mathbb{C})$.

Part III: Geometry

Let $m \geq d_1 > d_2 > \cdots > d_s \geq 0$ and let P be the subgroup of $G = GL_m(\mathbb{C})$ of block lower triangular matrices with block sizes $d_s, d_{s-1} - d_s, \ldots, d_1 - d_2$. It is natural to identify G/P and the (partial) *flag variety*

$$\mathrm{F}\ell^{d_1,\ldots,d_s} = \{E_1 \subseteq \cdots \subseteq E_s \subseteq E \mid \mathrm{codim}(E_i) = d_i\}.$$

If λ is a partition such that $\{d_1, \ldots, d_s\}$ is the set of lengths of columns of λ, then there is an imbedding

$$\begin{array}{ccccc} \mathrm{F}\ell^{d_1,\ldots,d_s} & = & G/P & \subseteq & \mathbb{P}^*(E^\lambda) \\ & & gP & \longmapsto & [g\varphi] \end{array}$$

where φ is the lowest weight vector in E^λ and $\mathbb{P}^*(E^\lambda)$ is the dual projective space of E^λ. The special cases $\lambda = (1^k)$ and $\lambda = (a)$ give the *Plücker imbedding* of the Grassmannian $\mathrm{Gr}^k(E) \subseteq \mathbb{P}^*(\bigwedge^k E)$ and the a-fold *Veronese imbedding* $\mathbb{P}^*(E) \subseteq \mathbb{P}^*(\mathrm{Sym}^a E)$, respectively.

A product of Plücker imbeddings gives $\mathrm{F}\ell^{d_1,\ldots,d_s} \subseteq \mathrm{Gr}^{d_1} E \times \cdots \times \mathrm{Gr}^{d_s} E \subseteq \mathbb{P}^*(\bigwedge^{d_1} E) \times \cdots \times \mathbb{P}^*(\bigwedge^{d_s} E)$, and only a little more argument is needed to show that the multihomogeneous coordinate ring of $\mathrm{F}\ell^{d_1,\ldots,d_s}$ is

$$S^\bullet(m; d_1, \ldots, d_s) = \frac{\mathrm{Sym}^\bullet(\bigwedge^{d_1} E) \otimes \cdots \otimes \mathrm{Sym}^\bullet(\bigwedge^{d_s} E)}{Q},$$

where Q is the ideal generated by the "quadratic relations". The algebra $S^\bullet(m; d_1, \ldots, d_s)$ is a subalgebra and $GL_m(\mathbb{C})$-submodule of the polynomial ring

$\mathbb{C}[Z_{ij} \mid 1 \le i \le n, 1 \le j \le m]$ under the natural action of $GL_m(\mathbb{C})$. There is also a (right) action of $GL_n(\mathbb{C})$ on $\mathbb{C}[Z_{ij}]$, and the *fundamental theorems of invariant theory* for $SL_n(\mathbb{C})$ can be encompassed in the statement

$$\mathbb{C}[Z_{ij}]^{SL_n(\mathbb{C})} = S^\bullet(m;n).$$

This fact is used to show that $S^\bullet(m;d_1,\dots,d_s)$ is a unique factorization domain.

Let $0 = F_0 \subseteq F_1 \subseteq \cdots \subseteq F_m = \mathbb{C}^m$ be a fixed flag of subspaces of $\mathbb{C}^m$ such that $\dim(F_i) = i$. For each partition λ which has at most $n-m$ rows and at most n columns (i.e. $\lambda \subseteq ((m-n)^n)$), the *Schubert variety*

$$\Omega_\lambda = \{V \in \mathrm{Gr}^n(\mathbb{C}^m) \mid \dim(V \cap F_{n+i-\lambda_i}) \ge i, \quad 1 \le i \le m-n\}$$

is an irreducible closed subvariety of dimension $|\lambda|$ in $\mathrm{Gr}^n(\mathbb{C}^m)$. The classes $[\Omega_\lambda]$, $\lambda \subseteq ((m-n)^n)$, form a basis of $H^*(\mathrm{Gr}^n(\mathbb{C}^m))$, and there is a ring homomorphism

$$\begin{array}{ccc} \text{symmetric functions} & \longrightarrow & H^*(\mathrm{Gr}^n(\mathbb{C}^m)) \\ s_\lambda(x) & \longmapsto & \begin{cases} [\Omega_\lambda], & \text{if } \lambda \subseteq ((m-n)^n), \\ 0 & \text{otherwise.} \end{cases} \end{array}$$

This fact is proved by analyzing carefully the intersection of three (generically placed) Schubert varieties Ω_λ, $\Omega_{(k)}$ and Ω_μ and showing that this gives the same multiplication rule for the classes $[\Omega_\lambda] \in H^*(\mathrm{Gr}^n(\mathbb{C}^m))$ as that satisfied by the corresponding Schur functions.

Let $\mathrm{F}\ell(m)$ be the flag variety of complete flags in $\mathbb{C}^m$, i.e. flags $E_1 \subseteq \cdots \subseteq E_m = \mathbb{C}^m$ such that $\dim(E_i) = i$. Let B be the subgroup of $GL_m(\mathbb{C})$ of upper triangular matrices, and let $\chi_i \colon B \to \mathbb{C}^*$ be the character of B such that $\chi_i(b)$ is the ith diagonal entry of b. Then

$$H^*(\mathrm{F}\ell(m)) = \frac{\mathbb{Z}[x_1,\dots,x_m]}{\langle e_1(x),\dots,e_m(x)\rangle},$$

where $x_i = -c_1(\chi_i)$ is the first Chern class of the line bundle $L_i = G \times_B \chi_i$ on G/B and $\langle e_1(x),\dots,e_m(x)\rangle$ is the ideal generated by the elementary symmetric functions $e_k(x) = s_{(1^k)}(x)$, $1 \le k \le m$.

For each $w \in S_m$ there is a *Schubert variety* X_w in $\mathrm{F}\ell(m)$, and the classes $[X_w]$ form a basis of $H^*(\mathrm{F}\ell(m))$. The dimension of X_w is the *length*, $\ell(w)$, of the permutation w and $X_v \subseteq X_w$ if and only if $v \le w$ in *Bruhat* order.

The inclusion $\mathbb{C}^m \subseteq \mathbb{C}^{(m+1)}$ (as the first m coordinates) induces an imbedding $\iota \colon \mathrm{F}\ell(m) \to \mathrm{F}\ell(m+1)$ and, in cohomology,

$$\begin{array}{cccc} \iota^*\colon & H^*(\mathrm{F}\ell(m+1)) & \longrightarrow & H^*(\mathrm{F}\ell(m)) \\ & x_i & \longmapsto & \begin{cases} x_i, & \text{if } i \le m, \\ 0, & \text{if } i > m, \end{cases} \\ & [X_w] & \longmapsto & [X_w], \qquad\qquad \text{for } w \in S_m, \end{array}$$

where $S_m \subseteq S_{m+1}$ as permutations which fix $m+1$. Fix i, $1 \le i \le m$, and let $Z = \{(E_\bullet, E'_\bullet) \mid E_\bullet, E'_\bullet \in \mathrm{F}\ell(m), E_j = E'_j \text{ for } j \ne i\}$. The projections p_1, p_2 onto the first and second components, respectively,

$$
\begin{array}{ccccc}
\begin{array}{c} Z \xrightarrow{p_2} \mathrm{F}\ell(m) \\ \downarrow{\scriptstyle p_1} \\ \mathrm{F}\ell(m) \end{array} & \text{give} &
\begin{array}{c} H^*(\mathrm{F}\ell(m)) \\ Q \\ [X_w] \end{array} &
\begin{array}{c} \xrightarrow{(p_1)_*(p_2^*)} \\ \longmapsto \\ \longmapsto \end{array} &
\begin{array}{c} H^*(\mathrm{F}\ell(m)) \\ \partial_i Q = \dfrac{Q - s_i Q}{x_i - x_{i+1}}, \\ \begin{cases} [X_{ws_i}], & \text{if } w(i) > w(i+1) \\ 0, & \text{if } w(i) < w(i+1), \end{cases} \end{array}
\end{array}
$$

in cohomology. If $w \in S_k$ then the *Schubert polynomial* $\mathfrak{S}_w$ is the unique homogeneous polynomial in $\mathbb{Z}[x_1, \ldots, x_k]$ such that $\mathfrak{S}_w$ is a representative of $[X_w] \in H^*(\mathrm{F}\ell(m))$ for all $m \geq k$.

Appendix A contains some further combinatorial techniques: evacuation, column insertion, dual Knuth equivalence, and the theory of keys and frank tableaux. *Appendix B* gives an efficient introduction to the basics of cohomology theory: definitions of pullback and pushforward, fundamental classes, Chern classes, intersections and Borel-Moore homology. Exercises for the reader are peppered throughout the main text, and the book concludes with a section giving brief answers to these exercises and some further notes and references.

Opinions

There has been a need for a book treating this material for some time, and the author has done an excellent job in choosing what to say and providing a concise yet thorough account of several "well known" topics for which it was hard to find a good reference. The treatment of many of the ideas of Lascoux and Schutzenberger will be very helpful to current and future researchers in this area. The brief introduction to intersection theory is to the point and exactly what is needed for those who need to study Schubert varieties and other similar algebraic varieties.

Although the style is very pleasant in its effort to make constructions natural and to illustrate the connections between various aspects of the material, this style also has its disadvantages. Sometimes one is not sure what has been actually proved at a given point, what is still left to be proved, exactly what the ingredients of the proof were, and exactly which parts of the proof were left to the reader or the exercises.

Especially since more than half the book is concerned with the theory of row insertion, it would be nice if there was more emphasis on the point that *this is not just a handy combinatorial game* for making slick proofs of identities. Row insertion and the Robinson-Schensted-Knuth (RSK) correspondence arise *naturally* in at least three algebraic/geometric settings, and only one of these is very briefly mentioned (Steinberg's realization of the RSK correspondence in terms of the decomposition of the unipotent variety). Two others are

(1) The fact that Knuth equivalence classes and Kazhdan-Lusztig cells are the same,
(2) The occurrence of the RSK-correspondence in the theory of crystals, i.e. canonical bases for quantum groups at $q = 0$.

Certainly it is not necessary for a book such as this one to treat all of these topics, but the point should be made that these combinatorial constructions do come up in several places and their study is beneficial to these other fields as well.

The material in this book should be of great interest to researchers and students interested in the representation theory and geometry of semisimple Lie groups. In some cases the $GL(n)$ focused presentation is very pleasantly indicative of the general semisimple group situation, and in other places it is quite difficult to see how these facts generalize, even in some of the cases where the generalization is known. Some parts of the text could have been treated slightly differently in order to make these connections and generalizations clearer, and the references could be more thorough in this direction. In particular, the following points need to be made:

(1) P. Littelmann [Li] has generalized the combinatorics of row insertion and the plactic algebra to all symmetrizable Kac-Moody Lie algebras,
(2) The theory of representations of GL_n as it is presented in this book can be done for all semisimple groups as in Janzten's book [Ja], and
(3) The theory of the cohomology of flag varieties for general Kac-Moody groups can be found in the work of Kostant-Kumar [KK].

The given construction of the irreducible representations of the symmetric group is one of the standard, but clumsy, ways of constructing these modules. There is no attempt made to relate this construction to other standard versions of this construction. These other versions and the relationships between them can be found in [Mac] Chapt. I, §7, Ex. 15 (a complete treatment, with proofs, of the representation theory of the symmetric group in $1\frac{1}{2}$ pages!).

There are two remarks to be made on notation.

(1) The term Young tableau is a bad one: it is overused and misused in the literature, and thus there is a general confusion about what this word really means. This book uses it to mean column strict tableau.
(2) It is very nice that most of the notations and definitions agree with that used in Macdonald's book on symmetric functions; it makes both books more useful. However, there are two crucial places, the meanings of "Jacobi-Trudi formula" and "Giambelli formula" [Mac, p. 61], where the terminology does differ.

In conclusion, this book is a book that every student and researcher in algebraic combinatorics, algebraic geometry, and/or representation theory should have on their shelves. The author has organized a wealth of information that was previously available only spread out in many parts of the literature. The book does not really make an effort to indicate the directions in which this field is growing, but it will be of great use to those who want a brief and well organized treatment of any of the topics included. It will be particularly useful for graduate students: the author has that magical ability for getting to the good stuff without getting the reader mired in preliminary "basics".

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