Alcove Walks and MV cycles Number Theory seminar Univ. of Melbourne 10.07.2013 ①

Langlands correspondence / Geometric Langlands

Representations of $G^\vee$ $\longleftrightarrow$ Geometry of G/K.

In our example:

$$G^\vee = PGL_3(\mathbb{C}) = GL_3(\mathbb{C}) / Z(GL_3(\mathbb{C})) = \frac{SL_3(\mathbb{C})}{Z(SL_3(\mathbb{C}))}$$

$$\cup\!\!\mid$$

$$T^\vee = \left\{ \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix} \right\}$$

$$\left\{ \begin{array}{l} \text{Irreducible representations} \\ \text{of } T^\vee \end{array} \right\} \longleftrightarrow \mathfrak{h}_{\mathbb{Z}} = \left\{ \mu^\vee = \mu_1\varepsilon_1^\vee + \mu_2\varepsilon_2^\vee + \mu_3\varepsilon_3^\vee \;\middle|\; \begin{array}{l} \mu_i \in \mathbb{Z} \\ \mu_1+\mu_2+\mu_3=0 \end{array} \right\}$$

$$y^{\mu^\vee} : T^\vee \to \mathbb{C}^\times \longleftarrow\!\!\shortmid \mu^\vee$$

$$\begin{pmatrix} x_1 & & \\ & x_2 & \\ & & x_3 \end{pmatrix} \mapsto x_1^{\mu_1} x_2^{\mu_2} x_3^{\mu_3}$$

$$\left\{ \begin{array}{l} \text{Irreducible} \\ \text{Representations} \\ \text{of } G^\vee \end{array} \right\} \longleftrightarrow \mathfrak{h}_{\mathbb{Z}}^+ = \left\{ \lambda^\vee = \lambda_1\varepsilon_1^\vee + \lambda_2\varepsilon_2^\vee + \lambda_3\varepsilon_3^\vee \;\middle|\; \begin{array}{l} \lambda_i \in \mathbb{Z} \\ \lambda_1+\lambda_2+\lambda_3=0 \\ \lambda_1 \geq \lambda_2 \geq \lambda_3 \end{array} \right\}$$

$$L(\lambda^\vee) \longleftarrow\!\!\shortmid \lambda^\vee$$

Then

$$\mathrm{Res}^{G^\vee}_{T^\vee}(L(\lambda^\vee)) = \bigoplus_{\mu^\vee \in \mathfrak{h}_{\mathbb{Z}}} L(\lambda^\vee)_{\mu^\vee}$$

where

$$L(\lambda^\vee)_{\mu^\vee} = \left\{ m \in L(\lambda^\vee) \;\middle|\; \begin{pmatrix} x_1 & & \\ & x_2 & \\ & & x_3 \end{pmatrix} \cdot m = x_1^{\mu_1} x_2^{\mu_2} x_3^{\mu_3} m \right\}$$

G/K, the loop Grassmannian

$$G = SL_3(\mathbb{C}((t))) \qquad \qquad U^- = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{pmatrix} \right\} \subseteq G.$$
$$\cup|$$
$$K = SL_3(\mathbb{C}[[t]])$$

Study G/K with

$$G = \bigsqcup_{\lambda^\vee \in \mathfrak{h}^+_{\mathbb{Z}}} K t_{\lambda^\vee} K \quad \text{and} \quad G = \bigsqcup_{\mu^\vee \in \mathfrak{h}_{\mathbb{Z}}} U^- t_{\mu^\vee} K$$

where $t_{\mu^\vee} = \begin{pmatrix} t^{-\mu_1} & 0 & 0 \\ 0 & t^{-\mu_2} & 0 \\ 0 & 0 & t^{-\mu_3} \end{pmatrix}$ if $\mu^\vee = \mu_1 \varepsilon_1^\vee + \mu_2 \varepsilon_2^\vee + \mu_3 \varepsilon_3^\vee$.

The <u>MV cycles</u> of type $\lambda^\vee$ and weight $\mu^\vee$ are the elements Z_b of

$$MV(\lambda^\vee)_{\mu^\vee} = \left\{ \begin{array}{c} \text{Irreducible components of} \\ \overline{K t_{\lambda^\vee} K \cap U^- t_{\mu^\vee} K} \end{array} \right\}$$

Mirković-Vilonen- .... -Lusztig-Casselman-Shalika say

$$\dim(L(\lambda^\vee)_{\mu^\vee}) = \mathrm{Card}(MV(\lambda^\vee)_{\mu^\vee}).$$

<u>Gaussent-Littelmann</u> Let $M_{\lambda^\vee\mu^\vee} = K t_{\lambda^\vee} K \cap U^- t_{\mu^\vee} K$.

(a) $M_{\lambda^\vee\mu^\vee} = \bigsqcup_{b \in B_{\lambda^\vee\mu^\vee}} C_b$ where $C_b = \left\{ x_p t_{\mu^\vee} K \;\middle|\; \begin{array}{l} p \in \mathcal{B}_{\lambda^\vee\mu^\vee} \text{ and} \\ p \text{ with labels removed} \\ \text{is } b \end{array} \right\}$

(b) $C_b = \mathbb{C}^{(\#\text{ of steps } \overset{-}{\to}{}^{+} \text{ in } b)} \times (\mathbb{C}^\times)^{(\#\text{ of steps } \overset{-}{\mapsto}{}^{+} \text{ in } b)}$

(c) $$MV(\lambda^\vee)_{\mu^\vee} \xleftrightarrow{\sim} \{ b \in B_{\lambda^\vee \mu^\vee} \mid \dim(b) \text{ is maximal} \}$$
$$Z_b = \overline{C_b} \longleftarrow\!\!\shortmid\ b$$

where $\dim(b) = (\#\text{steps } \overset{-\ +}{\rightarrow\!\!\!\!\!\!\mid\ } \text{ in } b) + (\#\text{steps } \overset{-\ +}{\mid\!\!\leftarrow} \text{ in } b)$.

A <u>labeled</u> <u>step</u> <u>of type j</u> is

$v \overset{-\ \ +}{\underset{d}{\longrightarrow}} vs_j$ or $v \overset{-\ \ +}{\underset{0}{\longleftarrow}} vs_j$ or $v \overset{-\ \ +}{\underset{d}{\mid\!\!\Longrightarrow}} vs_j$

$d \in \mathbb{C}$ $\qquad\qquad\qquad\qquad\qquad\qquad d \in \mathbb{C}^\times$

Let $\lambda^\vee \in \mathfrak{h}^+_{\mathbb{Z}}$ and fix a min. length path from 1 to $t_{w_0\lambda^\vee}$

$$t_{w_0\lambda^\vee} = s_{i_1} \cdots s_{i_\ell}$$

Let $P_{\lambda^\vee\mu^\vee} = \left\{ \begin{array}{l} \text{labeled folded walks of type } (i_1, \ldots, i_\ell) \\ \text{beginning at } 1 \text{ and with end hexagon } \mu^\vee \end{array} \right\}$

$B_{\lambda^\vee\mu^\vee} = \left\{ \begin{array}{l} \text{unlabeled folded walks of type } (i_1, \ldots, i_\ell) \\ \text{beginning at } 1 \text{ and with end hexagon } \mu^\vee \end{array} \right\}$

If $p = (\underbrace{p_1, \ldots, p_\ell}_{\text{steps}} \mid \underbrace{d_1, \ldots d_\ell}_{\text{labels}})$ is a labeled path

$$x_p = x_{\gamma_1}(d_1)\, x_{\gamma_2}(d_2) \cdots x_{\gamma_\ell}(d_\ell) \quad \text{where } x_{\gamma_j}(d_j) \in U^-$$

and

$\mathfrak{h}^{\gamma_1}, \ldots, \mathfrak{h}^{\gamma_\ell}$ is the sequence of hyperplanes corresponding to the steps of p.

Here

$$x_{\alpha_1}(f) = \begin{pmatrix} 1 & f & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad x_{\alpha_2}(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} \quad x_{\varphi}(f) = \begin{pmatrix} 1 & 0 & f \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_{\alpha_1}(f) = \begin{pmatrix} 1 & 0 & 0 \\ f & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad x_{-\alpha_2}(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & f & 1 \end{pmatrix} \quad x_{-\varphi}(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ f & 0 & 1 \end{pmatrix}$$

and

$$x_{\beta+k\delta}(c) = x_\beta(ct^k).$$

<u>Masters projects</u>

(e) Explain and write a clear exposition of Gaussent-Littelmann action of root ops with examples (say $L(2\alpha_1^\vee + 2\alpha_2^\vee)$)

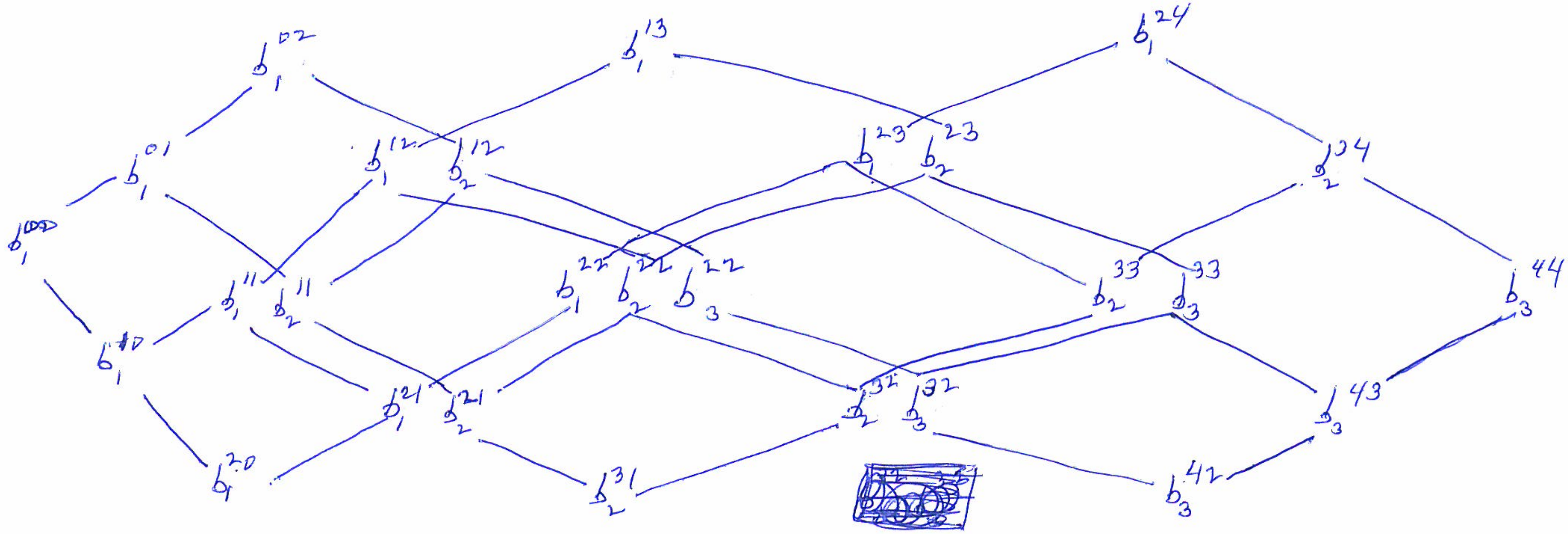
(a) Explain and write a clear exposition of Braverman-Finkelberg-Gaitsgory with examples (say $L(2\alpha_1^\vee + 2\alpha_2^\vee)$)

(b) Explain and write a clear exposition of Baumann-Gaussent with examples (say $L(2\alpha_1^\vee + 2\alpha_2^\vee)$)

(c) Explain and write a clear exposition of Baumann-Kamnitzer with examples (say $L(2\alpha_1^\vee + 2\alpha_2^\vee)$)

(d) Explain and write a clear exposition of Lauda-Vazirani with examples (say $L(2\alpha_1^\vee + 2\alpha_2^\vee)$)

Picture of $L(2\alpha_1^\vee + 2\alpha_2^\vee)$



dim $L(\lambda^\vee) = 27$

1 1 1
1 2 2 1
1 2 3 2 1
1 2 2 1
1 1 1

$\lambda^\vee = 2\alpha_1^\vee + 2\alpha_2^\vee$

$w_0 \lambda^\vee = -2\alpha_1^\vee - 2\alpha_2^\vee$

$\alpha_1^\vee$

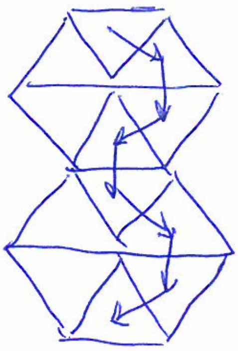
$\alpha_2^\vee$

$\lambda^\vee = 2(\alpha_1^\vee+\alpha_2^\vee)$

(1) $p = b_+$

b_1^{00} $wt = 0$

$dim = 0$

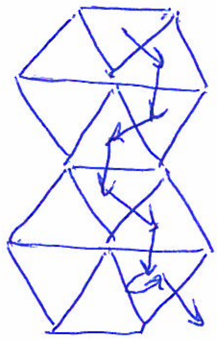


$$Z_b = \overline{\{t_{w_0\lambda^\vee} K\}}$$

(2) $p = \tilde{f}_1 b_+$

b_1^{10} $wt = -\alpha_1^\vee$

$dim = 1$

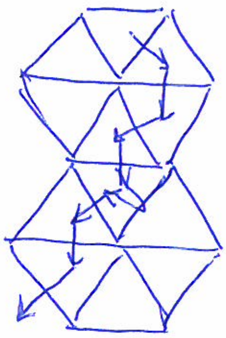


$$Z_b = \overline{\{x_{-\alpha_1-\delta}(d)\, t_{w_0\lambda^\vee+\alpha_1^\vee} K \mid d \in \mathbb{C}^\times\}}$$

(6) $p = \tilde{f}_2 b_+$

b_1^{01} $wt = -\alpha_2^\vee$

$dim = 1$

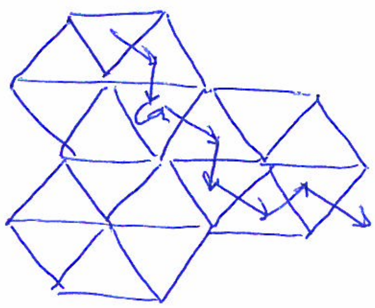


$$Z_b = \overline{\{x_{-\alpha_2-\delta}(d)\, t_{w_0\lambda^\vee+\alpha_2^\vee} K \mid d \in \mathbb{C}^\times\}}$$

(12) $p = \tilde{f}_1^{\,2} b_+$

b_1^{20} $wt = -2\alpha_1^\vee$

$dim = 2$

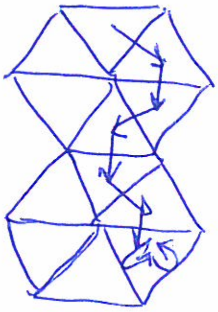


$$Z_b = \overline{\left\{x_{-\alpha_1}(d_1)\, x_{-\alpha_1+\delta}(d_2)\, t_{w_0\lambda^\vee+2\alpha_1^\vee} K \;\middle|\; \begin{array}{l} d_1 \in \mathbb{C}^\times \\ d_2 \in \mathbb{C} \end{array}\right\}}$$

(4) b_1 $\quad p = \check{f}_2 \check{f}_1 b_+$

$wt = -(\alpha_1^\vee + \alpha_2^\vee)$

$\dim = 2$

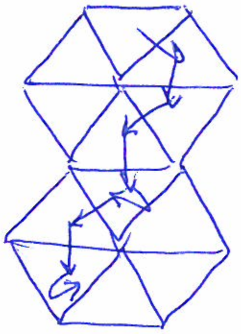


$$Z_b^2 = \overline{\left\{ x_{-\alpha_1-\delta}(d_1)\, x_{-\alpha_2-2\delta}(d_2)\, t_{w_0\lambda^\vee+\alpha_1^\vee+\alpha_2^\vee} K \;\middle|\; d_1, d_2 \in \mathbb{C}^\times \right\}}$$

(7) b_2^{11} $\quad p = \check{f}_1 \check{f}_2 b_+$

$wt = -(\alpha_1^\vee + \alpha_2^\vee)$

$\dim = 2$

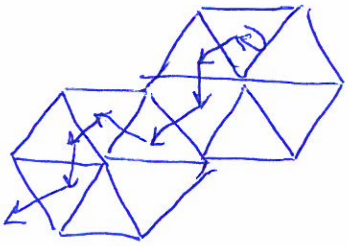


$$Z_b^2 = \overline{\left\{ x_{-\alpha_2-\delta}(d_1)\, x_{-\alpha_1-2\delta}(d_2)\, t_{w_0\lambda^\vee+\alpha_1^\vee+\alpha_2^\vee} K \;\middle|\; d_1, d_2 \in \mathbb{C}^\times \right\}}$$

(20) b_1^{02} $\quad p = \check{f}_2^{\,2} b_+$

$wt = -2\alpha_2^\vee$

$\dim = 2$

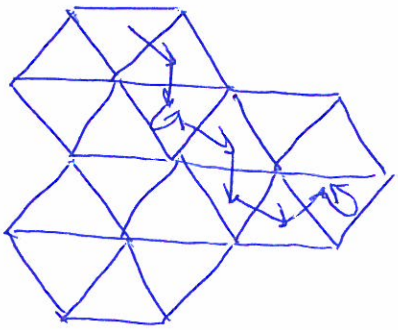


$$Z_b^2 = \overline{\left\{ x_{-\alpha_2}(d_1)\, x_{-\alpha_2+\delta}(d_2)\, t_{w_0\lambda^\vee+2\alpha_2^\vee} K \;\middle|\; d_1 \in \mathbb{C}^\times,\ d_2 \in \mathbb{C} \right\}}$$

(13) b_1^{21} $\quad p = \check{f}_2 \check{f}_1^{\,2} b_+ = \check{f}_1 \check{f}_2 \check{f}_1 b_+$

$wt = -2\alpha_1^\vee - \alpha_2^\vee$

$\dim = 3$



$$Z_b = \overline{\left\{ x_{-\alpha_1}(d_1)\, x_{-\alpha_1+\delta}(d_2)\, x_{-\alpha_2-3\delta}(d_3)\, t_{w_0\lambda^\vee-2\alpha_1^\vee-\alpha_2^\vee} K \;\middle|\; \begin{matrix} d_1, d_3 \in \mathbb{C}^\times \\ d_2 \in \mathbb{C} \end{matrix} \right\}}$$

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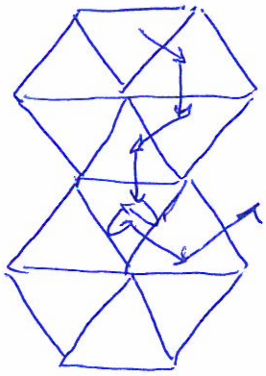
9 $p = \check{f}_1 \check{f}_2^2 b_+$

δ_2^{21}

$wt = -2\alpha_1^\vee - \alpha_2^\vee$

$dim = 3$



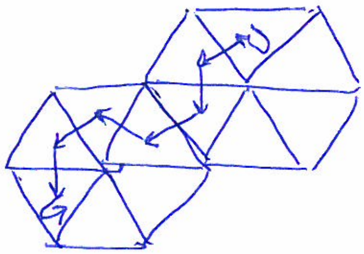
$$Z_b = \overline{\left\{ x_{-\alpha_2-\delta}(d_1)\, x_{\alpha_1-\delta}(d_2)\, x_{-\alpha_1}(d_3)\, t_{w_0\lambda^\vee + 2\alpha_1^\vee + \alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2 \in \mathbb{C}^\times \\ d_3 \in \mathbb{C} \end{array} \right\}}$$

21 $p = \check{f}_1 \check{f}_2^2 b_+$

δ_1^{12}

$wt = -\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 3$



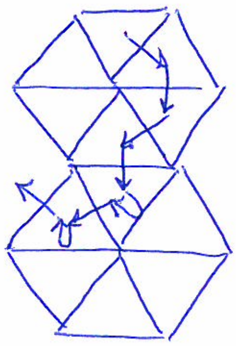
$$Z_b = \overline{\left\{ x_{-\alpha_2}(d_1)\, x_{-\alpha_2+\delta}(d_2)\, x_{-\alpha_1-3\delta}(d_3)\, t_{w_0\lambda^\vee + \alpha_1^\vee + 2\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_3 \in \mathbb{C}^\times \\ d_2 \in \mathbb{C} \end{array} \right\}}$$

8 $p = \check{f}_2^2 \check{f}_1 b_+ = \check{f}_2 \check{f}_1 \check{f}_2 b_+$

δ_2^{12}

$wt = -\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 3$



$$Z_b = \overline{\left\{ x_{-\alpha_2-\delta}(d_1)\, x_{-\varphi-2\delta}(d_2)\, x_{-\alpha_2}(d_3)\, t_{w_0\lambda^\vee + \alpha_1^\vee + 2\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2 \in \mathbb{C}^\times \\ d_3 \in \mathbb{C} \end{array} \right\}}$$

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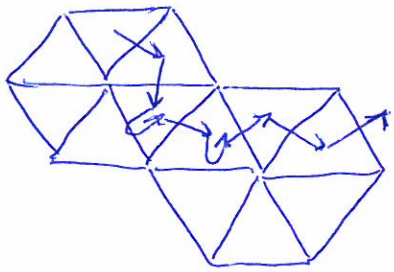
(15) $p = \hat{f}_1^{3} \hat{f}_2 b_+$

b_2^{31}

$wt = -3\alpha_1^\vee - \alpha_2^\vee$

$\dim = 4$



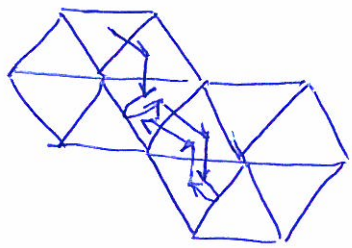
$$Z_b = \overline{\left\{ x_{-\alpha_1}(d_1)\, x_{-\varphi-\delta}(d_2)\, x_{-\alpha_1+\delta}(d_3)\, x_{-\alpha_1+\delta}(d_4)\, t_{w_0\lambda^\vee + 3\alpha_1^\vee + \alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2 \in \mathbb{C}^\times \\ d_3, d_4 \in \mathbb{C} \end{array} \right\}}$$

(14) $p = \hat{f}_2^{2} \hat{f}_1^{2} b_+$

b_1^{22}

$wt = -2\alpha_1^\vee - 2\alpha_2^\vee$

$\dim = 4$



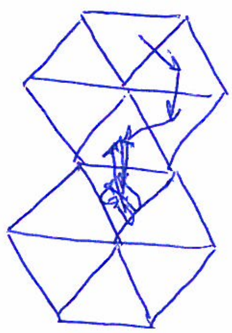
$$Z_b = \overline{\left\{ x_{-\alpha_1}(d_1)\, x_{-\alpha_2-2\delta}(d_2)\, x_{-\varphi-\delta}(d_3)\, x_{-\alpha_2-\delta}(d_4)\, t_{w_0\lambda^\vee + 2\alpha_1^\vee + 2\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2 \in \mathbb{C}^\times \\ d_3, d_4 \in \mathbb{C} \end{array} \right\}}$$

(10) $p = \hat{f}_1 \hat{f}_2^{2} \hat{f}_1 b_+ = \hat{f}_2 \hat{f}_1^{2} \hat{f}_2 b_+$

b_2^{22}

$wt = -2\alpha_1^\vee - 2\alpha_2^\vee$

$\dim = 4$



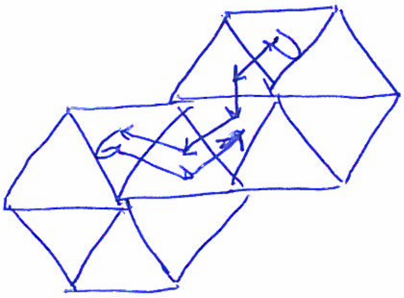
$$Z_b = \overline{\left\{ x_{-\alpha_2-\delta}(d_1)\, x_{-\alpha_1-\delta}(d_2)\, x_{-\alpha_2-\delta}(d_3)\, x_{-\varphi-\delta}(d_4)\, t_{w_0\lambda^\vee + 2\alpha_1^\vee + 2\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2, d_3 \in \mathbb{C}^\times \\ d_4 \in \mathbb{C} \end{array} \right\}}$$

(23) $p = \hat{f}_1^{2} \hat{f}_2^{2} b_+$

b_3^{22}

$wt = -2\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 4$



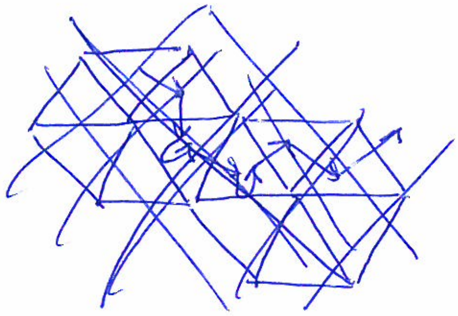
$$Z_b = \{ x_{\alpha_2}(d_1)\, x_{-\alpha_2+\delta}(d_2)\, x_{-\alpha_1-2\delta}(d_3)\, x_{-\alpha_1-\delta}(d_4)\, t_{w_0\lambda^\vee + 2\alpha_1^\vee + 2\alpha_2^\vee} K \mid d_1, d_3 \in \mathbb{C}^\times,\ d_2, d_4 \in \mathbb{C} \}$$

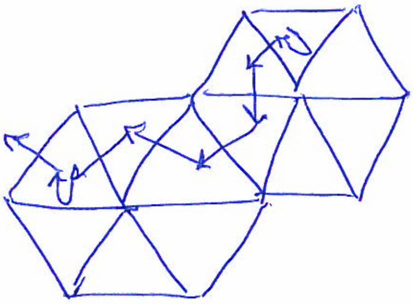
(22) $p = \hat{f}_2^{3} \hat{f}_1 b_+$

b_1^{13}

$wt = -\alpha_1^\vee - 3\alpha_2^\vee$

$dim = 4.$





$$\cancel{Z_b = \{ x_{-\alpha_1}(d_1)\, x_{-\varphi-\delta}(d_2)\, x_{-\alpha_1+\delta}(d_3)\, x_{-\alpha_1+2\delta}(d_4)\, t_{w_0\lambda^\vee + \alpha_1^\vee +}}$$

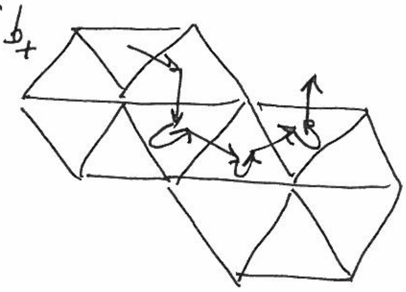
$$Z_b = \{ x_{-\alpha_2}(d_1)\, x_{-\alpha_2+\delta}(d_2)\, x_{-\varphi-\delta}(d_3)\, x_{-\alpha_2+2\delta}(d_4)\, t_{w_0\lambda^\vee + \alpha_1^\vee + 3\alpha_2^\vee} K \mid d_1, d_3 \in \mathbb{C}^\times,\ d_2, d_4 \in \mathbb{C} \}$$

(16) $p = \hat{f}_2 \hat{f}_1^{3} \hat{f}_2 b_+ = \hat{f}_1^{2} \hat{f}_2^{2} \hat{f}_1 b_+$

b_2^{32}

$wt = -3\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 5$



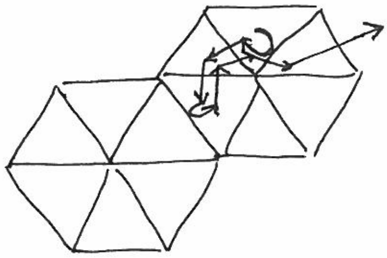
$$Z_b = \{ x_{-\alpha_1}(d_1)\, x_{-\varphi-\delta}(d_2)\, x_{-\alpha_1+\delta}(d_3)\, x_{-\alpha_2-2\delta}(d_4)\, x_{-\varphi}(d_5)\, t_{w_0\lambda^\vee + 3\alpha_1^\vee + 2\alpha_2^\vee} K \mid d_1, d_2, d_4 \in \mathbb{C}^\times,\ d_3, d_5 \in \mathbb{C} \}$$

(25) b_3^{32}

$\varphi = \tilde f_1^{3} \tilde f_2^{2} b_+$

$wt = -3\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 5$



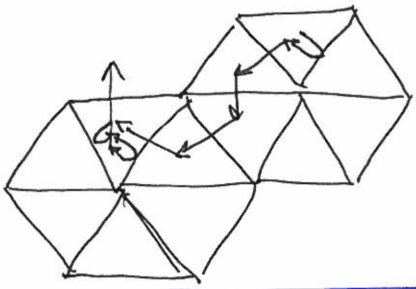
$z_6 = \{x_{-\alpha_2}(d_1)\, x_{-\alpha_1-\delta}(d_2)\, x_{\varphi}(d_3)\, x_{-\alpha_1}(d_4)\, X_{-\alpha_2+\delta}(d_5)\, t_{w_0\lambda^\vee+3\alpha_1^\vee+2\alpha_2^\vee} K \mid d_1, d_2 \in \mathbb{C},\ d_3, d_4, d_5 \in$

(24) b_2^{23}

$\varphi = \tilde f_2^{2} \tilde f_1^{2} \tilde f_2 b_+ = \tilde f_1 \tilde f_2^{3} \tilde f_1 b_+$

$wt = -2\alpha_1^\vee - 3\alpha_2^\vee$

$dim = 5$



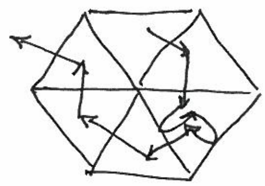
$z_6 = \{x_{-\alpha_2}(d_1)\, x_{\alpha_2+\delta}(d_2)\, x_{-\alpha_1-2\delta}(d_3)\, x_{-\alpha_2+\delta}(d_4)\, x_{\varphi}(d_5)\, t_{w_0\lambda^\vee+2\alpha_1^\vee+3\alpha_2^\vee} K \mid d_1, d_3, d_4 \in \mathbb{C}^\times,\ d_2, d_5 \in \mathbb{C}\}$

(17) b_1^{23}

$\varphi = \tilde f_2^{3} \tilde f_1^{2} b_+$

$wt = -2\alpha_1^\vee - 3\alpha_2^\vee$

$dim = 5$



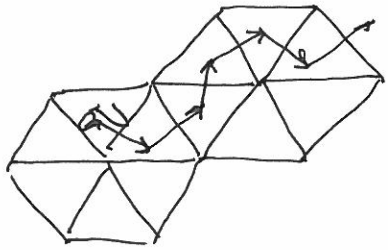
$z_6 = \{x_{-\alpha_1}(d_1)\, x_{-\alpha_2-\delta}(d_2)\, x_{-\alpha_2}(d_3)\, x_{-\varphi}(d_4)\, x_{-\alpha_2+\delta}(d_5)\, t_{w_0\lambda^\vee+2\alpha_1^\vee+3\alpha_2^\vee} K \mid d_1, d_2 \in \mathbb{C}^\times,\ d_3, d_4, d_5 \in \mathbb{C}\}$

(29) b_3^{42}

$\varphi = \tilde f_1^{4} \tilde f_2^{2} b_+$

$wt = -4\alpha_1^\vee - 2\alpha_2^\vee$

$dim = 6$



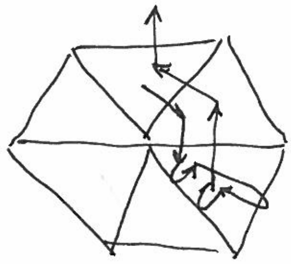
$z_6 = \{x_{-\alpha_2}(d_1)\, x_{-\alpha_1}(d_2)\, x_{-\alpha_1+\delta}(d_3)\, x_{-\varphi+\delta}(d_4)\, x_{-\alpha_1+2\delta}(d_5)\, x_{-\alpha_1+3\delta}(d_6)\, t_{w_0\lambda^\vee+4\alpha_1^\vee+2\alpha_2^\vee} K \mid d_1, d_2 \in \mathbb{C}^\times,\ d_3, d_4, d_5, d_6 \in$

(18)

b_2^{33} $p = \hat f_1 \hat f_2^3 \hat f_1^2 b_+ = \hat f_2^2 \hat f_1^3 \hat f_2 b_+$

$wt = -3\alpha_1^\vee - 3\alpha_2^\vee$

$\dim = 6$



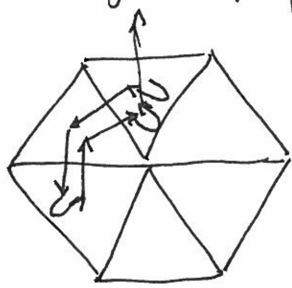
$Z_6 = \{x_{-\alpha_1}(d_1) x_{-\alpha_2-\delta}(d_2) x_{-\alpha_1}(d_3) x_{-\varphi}(d_4) x_{-\alpha_2}(d_5) x_{-\varphi+\delta}(d_6) t_{w_0\lambda^\vee+3\alpha_1^\vee+3\alpha_2^\vee} K \mid d_1, d_2, d_3 \in \mathbb{C}^\times,\ d_4, d_5, d_6 \in \mathbb{C}\}$

(26)

b_3^{33} $p = \hat f_1^2 \hat f_2^3 \hat f_1 b_+ = \hat f_2 \hat f_1^3 \hat f_2^2 b_+$

$wt = -3\alpha_1 - 3\alpha_2$

$\dim = 6$



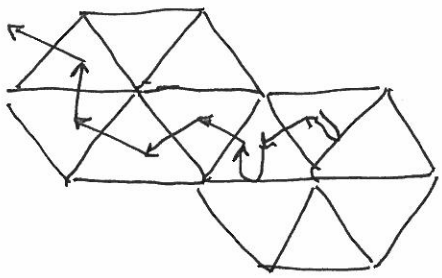
$Z_6 = \{x_{-\alpha_2}(d_1) x_{-\alpha_1-\delta}(d_2) x_{-\varphi}(d_3) x_{-\alpha_1}(d_4) x_{-\alpha_2}(d_5) x_{-\varphi+\delta}(d_6) t_{w_0\lambda^\vee+3\alpha_1^\vee+3\alpha_2^\vee} K \mid d_1, d_2, d_5 \in \mathbb{C}^\times,\ d_3, d_4, d_6 \in \mathbb{C}\}$

(27)

b_1^{24} $p = \hat f_2^4 \hat f_1^2 b_+$

$wt = -2\alpha_1^\vee - 4\alpha_2^\vee$

$\dim 6.$



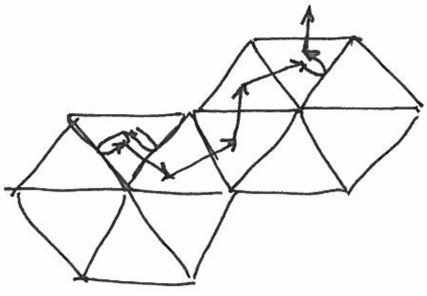
$Z_6 = \{x_{\alpha_2}(d_1) x_{-\varphi}(d_2) x_{-\alpha_2+\delta}(d_3) x_{-\alpha_2+2\delta}(d_4) x_{-\varphi+\delta}(d_5) x_{\alpha_2+3\delta}(d_6) t_{w_0\lambda^\vee+2\alpha_1^\vee+4\alpha_2^\vee} K \mid d_1, d_2 \in \mathbb{C}$ d_3, d_4, d_5, d

(30)

b_3^{43} $p = \hat f_2 \hat f_1^4 \hat f_2^2 b_+ = \hat f_1^3 \hat f_2^3 \hat f_1 b_+$

$wt = -4\alpha_1^\vee - 3\alpha_2^\vee$

$\dim = 7$



$Z_6 = \{x_{\alpha_2}(d_1) x_{\alpha_1}(d_2) x_{-\alpha_1+\delta}(d_3) x_{-\varphi+\delta}(d_4) x_{-\alpha_1+2\delta}(d_5) x_{-\alpha_2-\delta}(d_6) x_{-\varphi+2\delta}(d_7)) \mid d_1, d_2, d_6 \in \mathbb{C}^\times,\ d_3, d_4, d_5, d_7 \in \mathbb{C}$

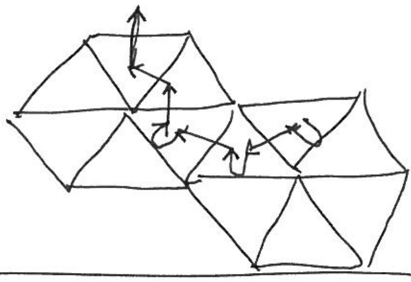
09.08.2013

8

28 $p = \hat{f}_2^{\,3}\check{f}_1^{\,3}\hat{f}_2 b_+ = \check{f}_1\check{f}_2^{\,4}\check{f}_1^{\,2} b_+$

b_2^{34} $wt = -3\alpha_1^\vee - 4\alpha_2^\vee$

$\dim = 7$

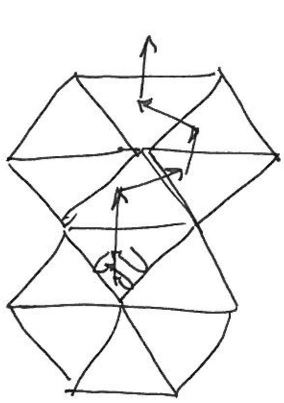


$$Z_b^2 = \left\{ x_{-\alpha_2}(d_1)\, x_{-\varphi}(d_2)\, x_{-\alpha_2+\delta}(d_3)\, x_{-\alpha_1}(d_4)\, x_{-\varphi+\delta}(d_5)\, x_{-\alpha_2+2\delta}(d_6)\, x_{\varphi+2\delta}(d_7)\, t_{w_0\lambda^\vee+3\alpha_1^\vee+4\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2, d_4 \in \mathbb{C}^\times \\ d_3, d_5, d_6, d_7 \in \mathbb{C} \end{array} \right\}$$

31 $p = \hat{f}_2^{\,2}\check{f}_1^{\,4}\hat{f}_2^{\,2} b_+ = \check{f}_1^{\,2}\check{f}_2^{\,4}\check{f}_1^{\,2} b_+$

b_3^{44} $wt = -4\alpha_1^\vee - 4\alpha_2^\vee$

$\dim = 8$



$$Z_b = \left\{ x_{-\alpha_2}(d_1)\, x_{-\alpha_1}(d_2)\, x_{-\alpha_2}(d_3)\, x_{-\varphi+\delta}(d_4)\, x_{-\alpha_1+\delta}(d_5)\, x_{-\varphi+2\delta}(d_6)\, x_{-\alpha_2+\delta}(d_7)\, x_{-\varphi+3\delta}(d_8) \;\; t_{w_0\lambda^\vee+4\alpha_1^\vee+4\alpha_2^\vee} K \;\middle|\; \begin{array}{l} d_1, d_2, d_3 \in \mathbb{C}^\times \\ d_4, d_5, d_6, d_7, d_8 \in \mathbb{C} \end{array} \right\}$$

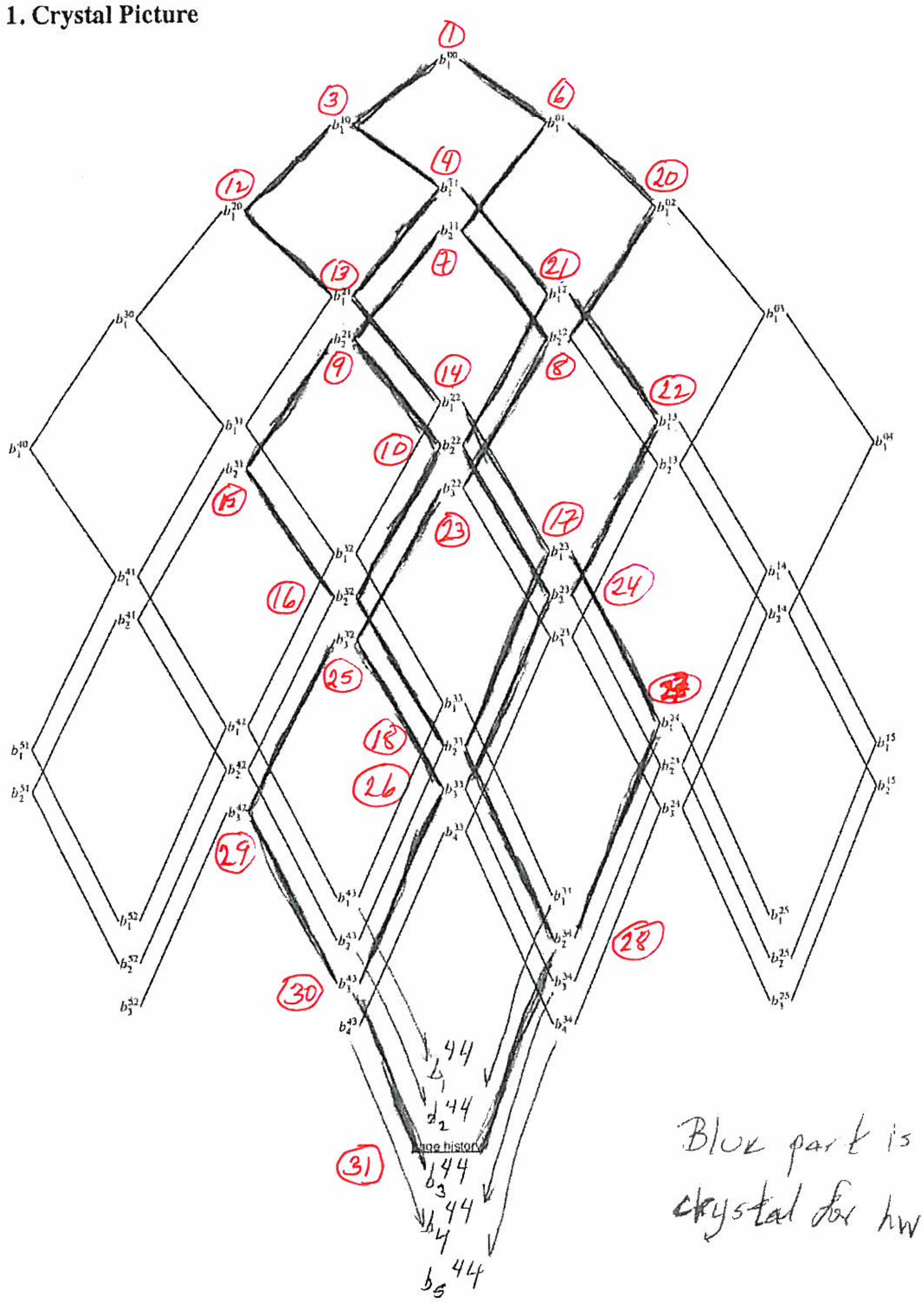
GHITZA

Crystal

Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville, VIC 3010 Australia
aram@unimelb.edu.au

Last update: 10 July 2012

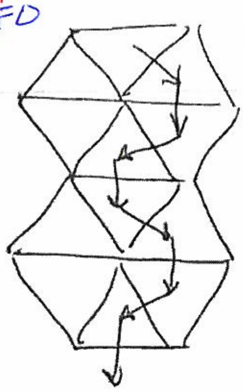
1. Crystal Picture



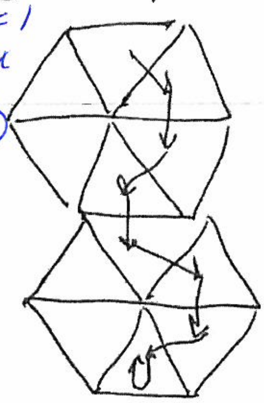
Blue part is the crystal for hw 2p.

Systematic

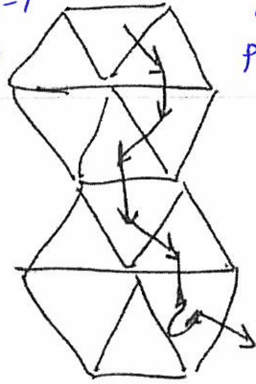
b_1^{00} (1)
dim = 0
$q = b_+$



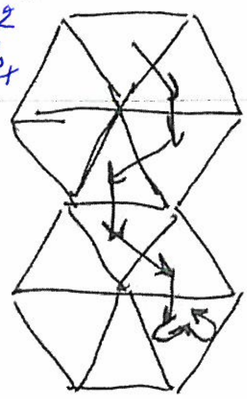
(2)
dim = 1
not max dim
see (4)



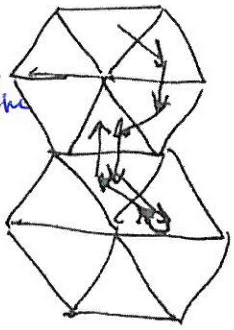
b_1^{10} (3)
dim = 1
$p = \hat{f}_1 b_+$



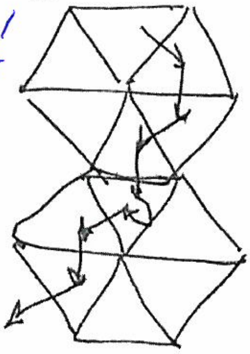
b_1^{11} (4)
dim = 2
$p = \hat{f}_2 \hat{f}_1 b_+$



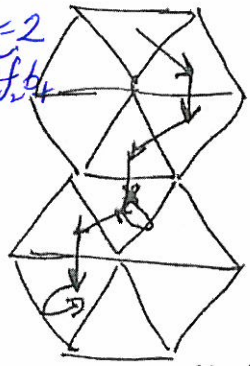
(5)
dim = 3
not max dim
see (10)



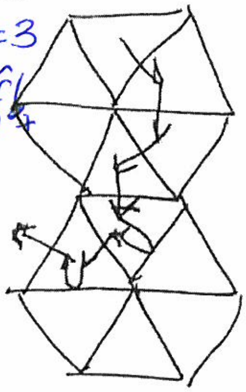
b_1^{01} (6)
dim = 1
$p = \hat{f}_2 b^+$



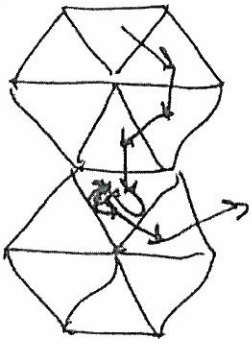
b_2^{11} (7)
dim = 2
$p = \hat{f}_1 \hat{f}_2 b_+$



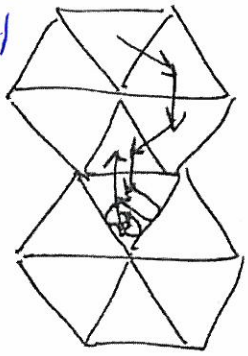
b_2^{12} (8)
dim = 3
$p = \hat{f}_2^{2} \hat{f}_1 b_+$



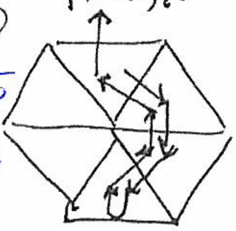
b_2^{21} (9)
dim = 3
$p = \hat{f}_1^{2} \hat{f}_2 b^+$



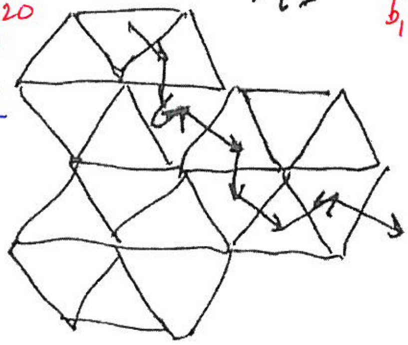
b_2^{22} (10)
dim = 4



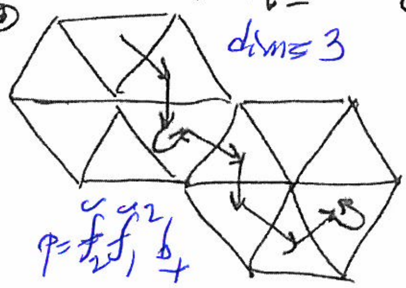
(11)
dim = 5
not max
dim see
(26)



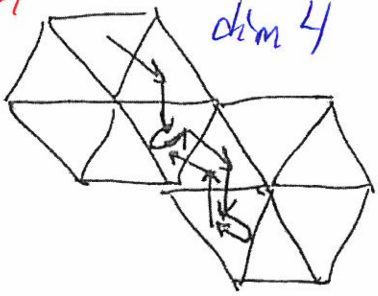
(12) b_1^{20}
dim = 2
$p = \hat{f}_1^{2} b_+$



b_1^{21} (13)
dim = 3
$p = \hat{f}_2 \hat{f}_1^{2} b_+$



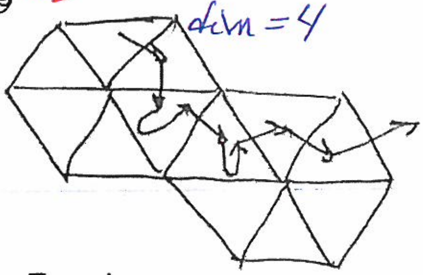
(14) b_1^{22}
dim 4



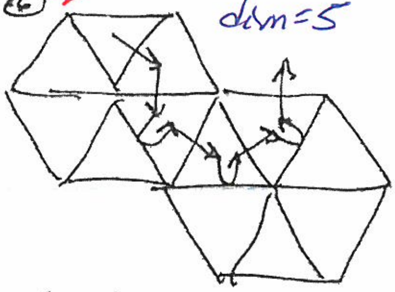
20.03.2012

(4)

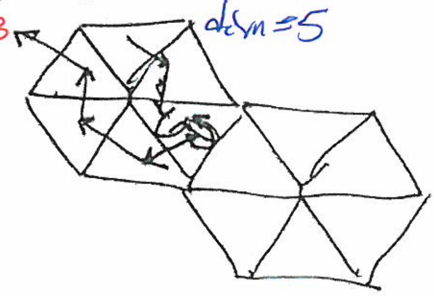
(15) b_2^{31} $-3\alpha_1-\alpha_2$ dim = 4



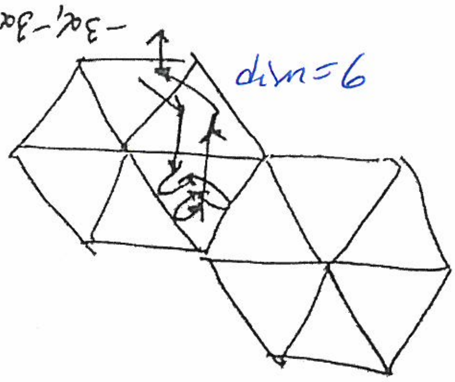
(16) b_2^{32} $-3\alpha_1-2\alpha_2$ dim = 5



(17) b_1^{23} $-2\alpha_1-3\alpha_2$ dim = 5



(18) b_2^{33} $-3\alpha_1-3\alpha_2$ dim = 6

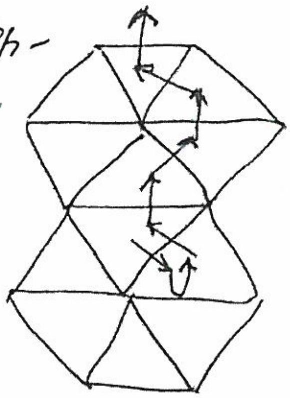


(19) $-4\alpha_1-4\alpha_2$

dim = 7

not max dim

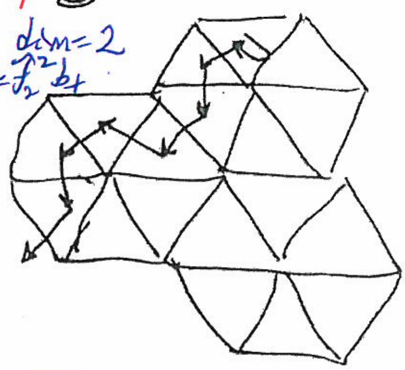
(see (37))



(20) b_1^{02} $-2\alpha_2$

dim = 2

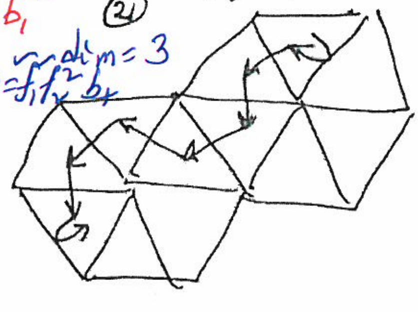
$p = f_2^2 b_+$



(21) b_1^{12} $-\alpha_1-2\alpha_2$

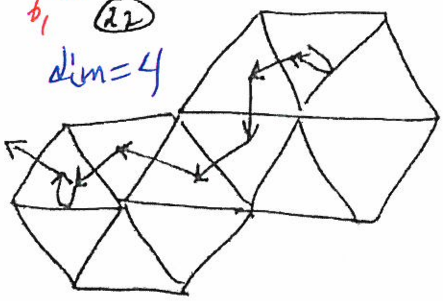
dim = 3

$p = f_1 f_2^2 b_+$

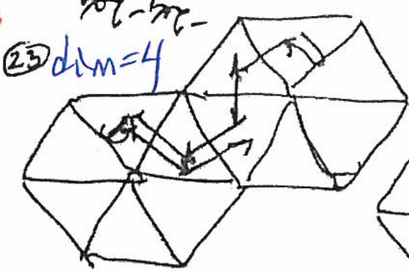


(22) b_1^{13} $-\alpha_1-3\alpha_2$

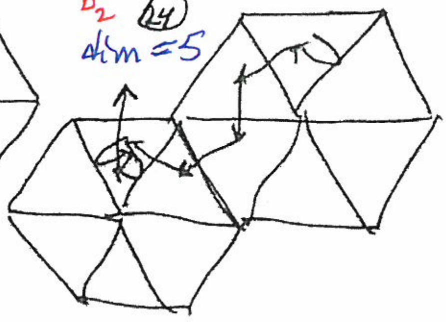
dim = 4



(23) b_3^{22} $-2\alpha_1-2\alpha_2$ dim = 4

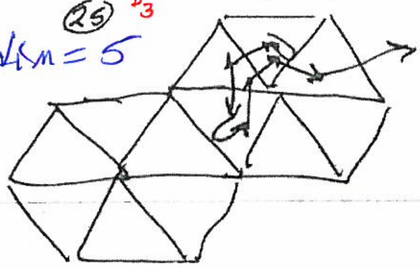


(24) b_2^{23} $-2\alpha_1-3\alpha_2$ dim = 5

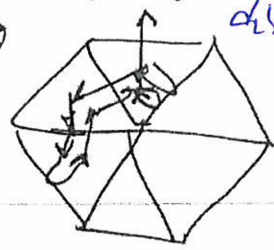


20.03.2012 (5)

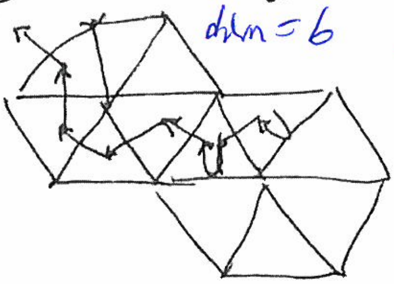
(25) $dim = 5$ b_3^{32} $-3\alpha_1-2\alpha_2$



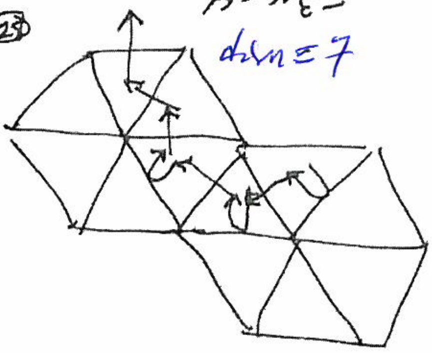
(26) $dim = 6$ b_3^{33} $-3\alpha_1-3\alpha_2$



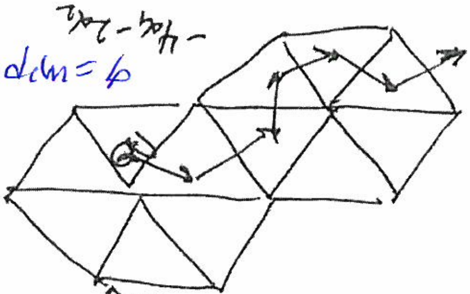
(27) $dim = 6$ b_1^{24} $-2\alpha_1-4\alpha_2$



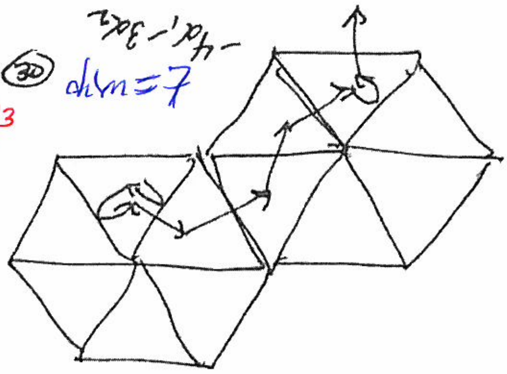
(28) $dim = 7$ b_2^{34} $-3\alpha_1-4\alpha_2$



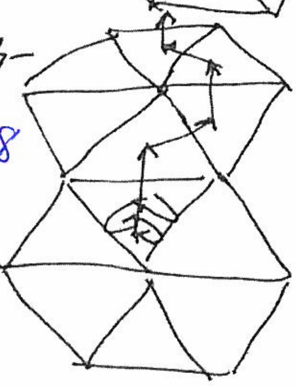
(29) $dim = 6$ b_3^{42} $-4\alpha_1-2\alpha_2$



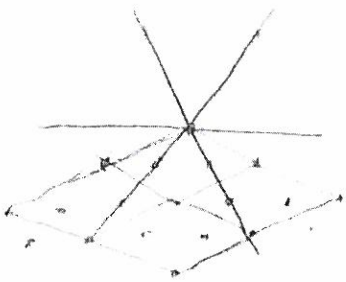
(30) $dim = 7$ b_3^{43} $-4\alpha_1-3\alpha_2$



(31) $dim = 8$ b_3^{44} $-4\alpha_1-4\alpha_2$



3·4



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