The configuration space

Let

$$\mathring{I} \subseteq \mathfrak{h}^*/\mathbb{R}\delta \quad \text{be identified with } \mathfrak{h}^*_{\mathbb{R}} + \mathbb{R}_{>0}\Lambda_0 \subseteq \mathfrak{h}^*_{\mathbb{R}} + \mathbb{R}\Lambda_0$$

For $\beta \in \mathring{R}$ and $m \in \mathbb{Z}$ let

$$\tilde{H}_{\beta+m\delta,k} = \{\mu + r\Lambda_0 \in \mathfrak{h}^*_{\mathbb{R}} + \mathbb{R}\Lambda_0 \mid \langle \mu + r\Lambda_0, (\beta+m\delta)^\vee \rangle = k\}$$

and define

$$\hat{Y} = \left(\mathfrak{h}^*/\mathbb{R}\delta + i\mathring{I} - \bigcup_{\substack{\beta \in \mathring{R} \\ m,k \in \mathbb{Z}}} \hat{H}_{(\beta+m\delta)^\vee,k} + i\,\hat{H}_{(\beta+m\delta)^\vee,0} \right) \times \mathbb{C}^\times$$

with base point

$$(\rho_c + i\rho_c, z_0) \quad \text{where} \quad \rho_c = \frac{1}{1000}\rho \quad \text{and} \quad z_0 = \frac{1}{1000}$$

PICTURES

The double affine Weyl group

$$\hat{W} = \{ q^k X^\mu w Y^\nu \mid \mu \in \mathfrak{h}^*_{\mathbb{Z}},\ w \in W_0,\ \nu \in \mathfrak{h}_{\mathbb{Z}},\ k \in \mathbb{Z} \}$$

with

$$X^\mu X^\gamma = X^{\mu+\gamma}, \qquad Y^\nu Y^\sigma = Y^{\nu+\sigma}$$

$$w X^\mu w^{-1} = X^{w\mu}, \qquad w Y^\nu w^{-1} = Y^{w\nu}$$

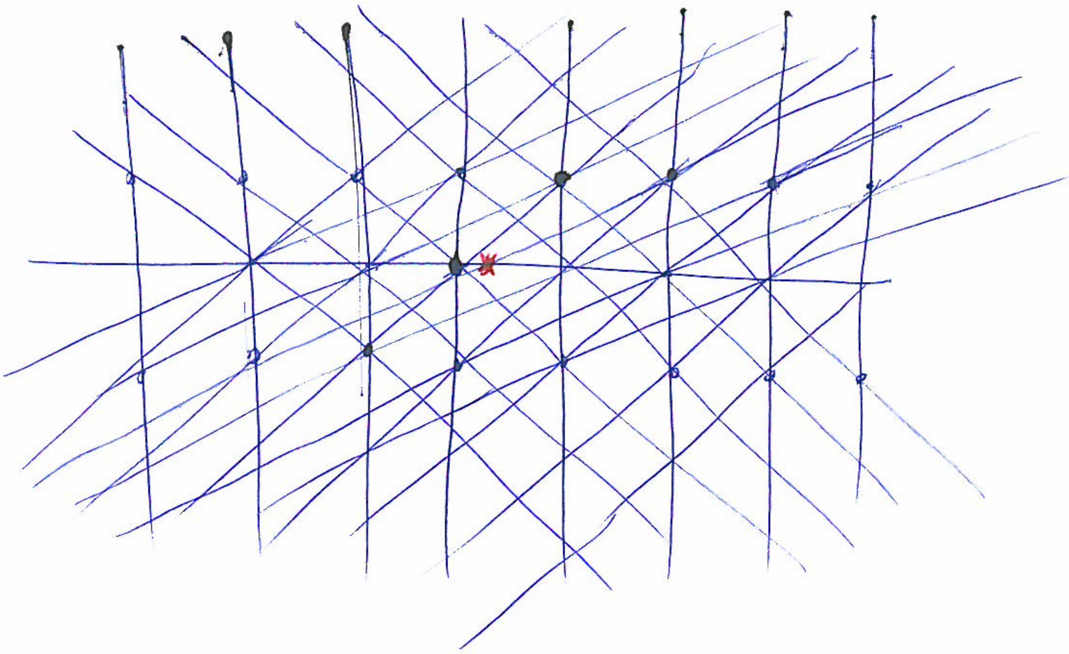
$$q \in Z(\hat{W}) \quad \text{and} \quad X^\mu Y^\nu = q^{\langle \mu, \nu \rangle} Y^\nu X^\mu$$

Let

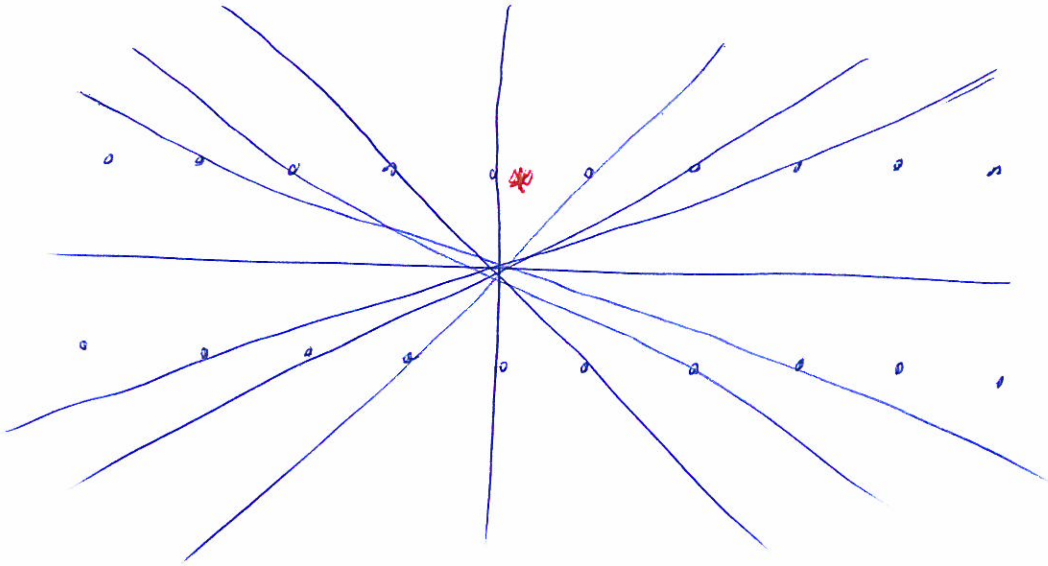
$$q = X^\delta = Y^{-d}.$$

Semivar 13.10.2013 (2)

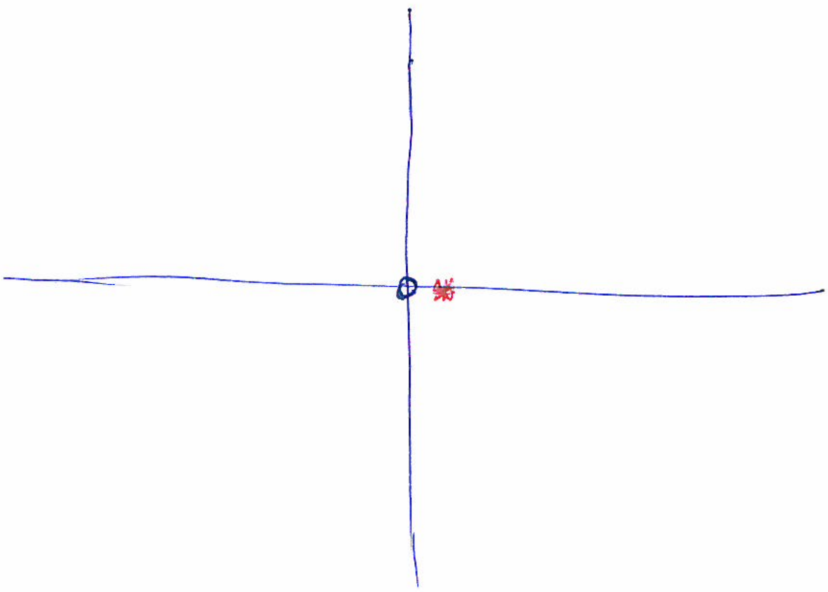
$\mathfrak{g}^*/RS$



$i\Lambda_0$



$\mathbb{C}^*$



The <u>double affine braid group</u> $\mathcal{B}_{\tilde{W}}$ is generated by $T_0, T_1, \ldots, T_n$ and X_{α_j} for $j \in \{1, 2, \ldots, n\}$ and X_δ with

$$\underbrace{T_i T_j \cdots}_{m_{ij}} = \underbrace{T_j T_i \cdots}_{m_{ij}} \quad \text{for } i \neq j$$

$$X_\delta \in Z(\mathcal{B}_{\tilde{W}}), \quad X_\beta X_\gamma = X_{\beta+\gamma} \quad \text{for } \beta, \gamma \in \sum_{j=1}^{n} \mathbb{Z}\alpha_j$$

$$T_j X_\beta = X_\beta T_j, \quad \text{if } \langle \beta, \alpha_j^\vee \rangle = 0$$

$$T_j X_\beta \cancel{T_j} = X_{s_j\beta}, \quad \text{if } \langle \beta, \alpha_j^\vee \rangle = 1$$

$\tilde{W}$ acts on $\tilde{Y}$ by ....

<u>Theorem</u> The map

$$\begin{aligned} \pi_1(\tilde{Y}/\tilde{W}) &\longrightarrow \mathcal{B}_{\tilde{W}} \\ T_j(t) &\longmapsto T_j, \quad \text{for } j \in \{0, 1, \ldots, n\} \\ X_{\alpha_j}(t) &\longmapsto X_{\alpha_j} \\ X_\delta(t) &\longmapsto X_\delta \end{aligned}$$

is an isomorphism, where

see preferred way to write paths on bottom of p.8.

$\tilde{W}$ acts on

(1) Ion $\tilde{Y}$ by ...

(2) Sahi-Ion $\tilde{\tilde{Y}}$ by ....

(3) Ram $\mathfrak{h}^*$ by ...

(4) Van der Lek Ω by ...

(5) Kac-Peterson Y by ...

First let

$$s_0 = Y^{\varphi^\vee} s_\varphi$$

(1) Ion $\tilde{Y} = \left(\left(\mathfrak{h}^*_R / R\delta + i\underline{\mathcal{I}} \right) - \bigcup_{\substack{\beta\in R \\ k, m\in\mathbb{Z}}} H_{(\beta+m\delta)^\vee, k} + i H_{(\beta+m\delta)^\vee, 0} \right) \times \mathbb{C}^\times$.

with

$$s_j(\hat{v}_1 + i\hat{v}_2, z) = (s_j\hat{v}_1 + i s_j \hat{v}_2, z) \quad \text{for } j\in\{1, 2, \dots, n\}.$$

$$s_0(\hat{v}_1 + i\hat{v}_2, z) = \left(s_0\hat{v}_1 + i s_0\hat{v}_2, z e^{\frac{i\pi}{2}\left(\langle\mu,\theta\rangle - \frac{r}{s}\langle\nu,\theta\rangle\right)}\right)$$

$$X^\beta(\mu + r\lambda_0 + i(\nu + s\lambda_0), z) = \left(\beta + \mu + r\lambda_0, i(\nu + s\lambda_0); z e^{\frac{i\pi}{2}\frac{1}{s}\langle\nu,\beta\rangle}\right)$$

$$X^\delta(\hat{v}_1 + i\hat{v}_2, z) = \left(\hat{v}_1 + i\hat{v}_2, z e^{\frac{i\pi}{2}\cdot 2}\right)$$

$$s_j = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s_j & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ \mu \\ r \end{pmatrix} \qquad s_0 = \begin{pmatrix} 1 & \theta^\vee & -\frac{1}{2}\langle\theta^\vee, \theta^\vee\rangle \\ 0 & s_\theta & \theta^\vee \\ 0 & 0 & 1 \end{pmatrix} = s_{-\theta+\delta}$$

For X_β and X^δ see other side