Arun's TEX notes 2015

Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville VIC 3010 Australia
aram@unimelb.edu.au

October 29, 2015

1 Problems to work on

1.1 Crystals

(1) S_n crystals

(2) Virasoro crystals

(3) Fusion

(4) Schubert crystals

(5) Properly work through Littelmann's Verma bases paper

1.2 Complex reflection groups

(1) Chevalley-Shephard-Todd

(2) Revised classification by "affine" complex reflection groups

(3) equivalence of categories to p-compact groups

(4) $K_T(G/B)$ via moment graphs

(5) Set Bott towers and Bruhat order for $H_T(G/B)$

1.3 Quantum groups at roots of 1

(1) Fock space straightening laws

(2) translation functor algebra action on Fock space

(3) tensor product crystals (follow sketch from an email to Martina Lanini in December 2012)

(4) Verma module decomposition numbers

AMS Subject Classifications: Primary ????; Secondary ????.

1.4 Affine Lie algebras

(1) The quantum group is a fake

(2) Critical level decomposition numbers for category $\mathcal{O}$

(3) The squish map to finite dimensional representations of level 0

1.5 Yangians

(1) Write the affine quantum group to Yangian specialisation

(2) Set up the elliptic Yangian (follow Toledano Laredo and Gautam)

(3) A path model basis of finite dimensional tame representations of Yangians (follow Nazarov-Tarasov and Ram)

(4) Set up the Bethe Ansatz as a weight space analysis of all finite dimensional Yangian modules

1.6 Braid groups

(1) Classical type DAArts as braids

(2) DAArts as fundamental groups, the affine version (all levels)

(3) DAArts as fundamental groups, the elliptic version

(4) DAArts as fundamental groups, the single level version

1.7 Combinatorics of affine flag varieties

(1) A refined alcove walk model for the twisted case $G^\sigma = \{g \in G \mid \sigma(g) = g\}$ (i.e. rewrite Parkinson-Ram-Schwer to handle the twisted case)

(2) A refined alcove walk model with the central extension in $\{1\} \to Z \to \tilde{G} \to G \to \{1\}$ (i.e. rewrite Parkinson-Ram-Schwer to handle the central extension)

(3) A refined alcove walk model for affine Springer fibers (i.e. rewrite Parkinson-Ram-Schwer to handle the affine Springer Fiber case)

(4) Rewrite Peterson's notes on quantum cohomology

1.8 Tantalizers

(1) Work out the Jantzen determinants for $\cdot \otimes V$ in classical type

(2) Provide a grading (KLR version) of affine BMW algebras

(3) Write a proper proof that the irreducible finite dimensional modules for affine BMW algebras are indexed by multisegments with $k, k-2, \ldots$ boxes

(4) Work out the combinatorics for classical type analogous to the Misra-Miwa Fock space.

1.9 Hecke algebras

(1) Match the classification of (a) affine root systems (Macdonald) (b) affine Lie algebras (Kac) (c) reductive groups over local fields (Bruhat-Tits) – include the "non-reduced" cases (twistings) and the "lattices" (central extensions)

(2) Explain how the DAHA and Macdonald polynomials for affine root systems of classical type are obtained "by restriction" from type $(C_n^\vee, C_n)$.

(3) Explain how DAHA "specializes" to the graded affine Hecke algebra and to the rational Cherednik algebra

(4) Explain how DAHA acts on the elliptic cohomology of a finite Springer fiber

(5) Provide a "fundamental Langlands diagram" for Macdonald polynomials

(6) Describe the transition matrix between $P_\lambda(q, q^k)$ and $P_\lambda(q, q^{k+1})$ (this is an analogue of the Kostka-Foulkes matrix which should be informed by a geometric Atiyah-Bott-Lefschetz fixed point localization formula)

(7) Read, streamline, and tighten Martha Yip's PhD thesis (in particular, sort out the positivity properties of the Littlewood-Richardson rule that she gives)

1.10 Quiver varieties

(1) Work out the projective convolution over $\mathbb{F}_q$ "à la Ringel"

(2) Match up points of quiver variety orbit closures and MV-cycles

(3) Complete the study of the Kashiwara-Saito example and the analogous phenomena in $A_1^{(1)}$ and D_4 and other "imaginary root" cases

(4) Work out the nonsimply laced KLR representations by "folding" of the Dynkin diagram

1.11 Schubert varieties

(1) Analyze the tangent spaces at T-fixed points following Gaussent and Manivel

(2) Describe the fibers of the Bott-Samelson to Schubert vareity squish map with path model tools

(3) Describe the Schubert basis in $Ell_T(G/B)$ and $\Omega_T(G/B)$ by setting up an appropriate Poincare duality statement and using a support and orthogonal basis condition.

(4) Write generators and relations for $\Omega_T(X)$ where X is a Bott tower.

References

[AB] M.F. Atiyah and R. Bott, *The moment map and equivariant cohomology*, Topology **23** (1984) 128, MR 0721448.

[Ad] J.F. Adams, *Stable homotopy and generalised homology*, Univ. of Chicago Press, 1974.

[AF] T. Arakawa, P. Fiebig, *On the restricted Verma modules at the critical level*, ???.

[Bax] R. Baxter, *Exactly solved models in statistical mechanics,* Reprint of the 1982 original. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], London, 1989. xii+486 pp. ISBN: 0-12-083182-1 MR0998375 (90b:82001)

[BD] Y. Billig and M. Dyer, *Decompositions of Bruhat type for the Kac-Moody groups*, Nova J. Algebra Geom. **3** no. 1 (1994), 11–31.

[BS0] J. Bernstein and O. Schwarzman, *Chevalley's theorem for complex crystallographic Coxeter groups*, Funktsional. Anal. i Prilhozen. **12** (1978), 79-80 [Russian], English translation: Funct. Anal. Appl. **12** (1978), 308-309 (1979).

[BS1] J. Bernstein and O. Schwarzman, *Complex crystallographic Coxeter groups and affine root systems*, J. Nonlinear Math. Physics, **13** no. 2 (2006), 163-182. MR?????

[BS2] J. Bernstein and O. Schwarzman, *Chevalley's theorem for the complex crystallographic groups*, J. Nonlinear Math. Physics, **13** no. 3 (2006), 323-351. MR?????

[BiLa] S. Billey and V. Lakshmibai, *????*, Birkhauser, ????, MR ????.

[Bo] , A. Borel, *Sur la cohomologie des espaces fibrés principaux et des espaces homogènes de groupes de Lie compacts*, Ann. of Math. (2) **57** (1953) 115207, MR0051508.

[BL] L. Borisov and A. Libgober, *Elliptic genera of singular varieties*, Duke Math. J. **116** (2003) 319351, MR1953295

[Bou] N. Bourbaki. *Éléments de mathématique Algèbres*, ?????????

[Bou2] N. Bourbaki. *Groups et Algèbres de Lie*, Ch. 4-6, ????

[BE1] P. Bressler and S. Evens, *The Schubert calculus, braid relations, and generalized cohomology*, Trans. Amer. Math. Soc. **317** (1990), 799- 811.

[BE2] P. Bressler and S. Evens, *Schubert calculus in complex cobordism*, Trans. Amer. Math. Soc. **331** (1992), 799- 813.

[Bri] M. Brion, *Lectures on the Geometry of flag varieties*, http://www-fourier.ujf-grenoble.fr/ mbrion/lecturesrev.pdf

[BT] F. Bruhat and J. Tits, *Groupes réductifs sur un corps local: I. Données radicielles valuées*, Publications Mathématiques de l'Institut des Hautes Études Scientifiques, no. **41** (1972).

[Ca] R. Carter, *Finite groups of Lie type – Conjugacy classes and complex characters*, Wiley ???.

[Ch1] I. Cherednik, *Diagonal coinvariants and double affine Hecke algebras*, Int. Math. Res. Not. **16** (2004), 769–791. MR2036955. arXiv:math.QA/0305245.

[C03] I.V. Cherednik, *Double affine Hecke algebras and difference Fourier transforms*, Invent. Math. **152** (2003), no. 2, 213–303. MR1974888, arXiv:math.QA/0110024.

[Ch] W.-L. Chow, *Algebraic varieties with rational dissections*, Proc. Nat. Acad. Sci. U.S.A. **42** (1956) 116119, MR0078006.

[CG] N. Chriss and V. Ginzburg, *Representation theory and complex geometry*, Birkhäuser Boston, Inc., Boston, MA, 1997. x+495 pp. ISBN: 0-8176-3792-3, MR1433132 and MR2838836

[CPZ] B. Calmès, V. Petrov, K. Zainoulline, *Invariants, torsion indeices and oriented cohomology of complete flags*, arxiv:0905.1341

[GN] J. de Gier and A. Nichols, *The two-boundary Temperley-Lieb algebra*, J. Algebra **321** (2009) 1132–1167. arXiv:math.RT/0703338

[Deo] V.V. Deodhar, *On some geometric aspects of Bruhat orderings. I. A finer decomposition of Bruhat cells*, Invent. Math. **79** (1985) 499-511.

[Dy] ????. Dyer, *Comparison between cohomology theories*, in J.F. Adams, *A student's guide to algebraic topology*, ????????????

[EG] P. Etingof and V. Ginzburg, *Symplectic reflection algebras, Calogero-Moser space, and deformed Harish-Chandra homomorphism*, Invent. Math. **147** (2002), no. 2, 243–348. MR1881922, arXiv:math.AG/0011114.

[Fa1] L.D. Faddeev, *How the algebraic Bethe ansatz works for integrable models*, Symtries quantiques (Les Houches, 1995), 149–219, North-Holland, Amsterdam, 1998. MR1616371 (2000b:82010)

[Fa2] L.D. Faddeev, *Algebraic aspects of the Bethe ansatz*, Internat. J. Modern Phys. A **10** (1995), no. 13, 1845–1878. MR1328109 (96c:82017)

[Fu] W. Fulton, *Intersection Theory*, Second edition, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 2. Springer-Verlag, Berlin, 1998. xiv+470 pp. ISBN: 3-540-62046-X; 0-387-98549-2, MR1644323

[Ga] N. Ganter, *Notes on the Weyl character formula and elliptic cohomology*, Univ. of Melbourne, 2010.

[GKV] V. Ginzburg, M. Kapranov and E. Vasserot, *Elliptic algebras and equivariant elliptic cohomology I*, arXiv:q-alg/9505012.

[Go] I. Gordon, *On the quotient ring by diagonal invariants*, Invent. Math. **153** (2003), 503–518. MR2000467, arXiv:math.RT/0208126.

[GKM] M. Goresky, R. Kottwitz, R. MacPherson, *Equivariant cohomology, Koszul duality, and the localization theorem*, Invent. Math. 131 (1998), no. 1, 2583, MR1489894.

[GR] S. Griffeth and A. Ram, *Affine Hecke algebras and the Schubert calculus*, European J. Combinatorics **25** (2004) 1263-1283.

[Gr] I. Grojnowski, *Delocalised equivariant elliptic cohomology*, 1994, available from http://www.dpmms.cam.ac.uk/∼groj/papers.html.

[G] A. Grothendieck, *Sur quelques propriétés fondamentales en théorie des intersections*, Séminaire Claude Chevalley **3** (1958), Exposé No. 4, 36 p.

[H06] M. Haiman, *Cherednik algebras, Macdonald polynomials and combinatorics*, Proc. ICM, Madrid 2006, Vol. **III**, 843-872.

[HHH] M. Harada, A. Henriques, T. Holm, *Computation of generalized equivariant cohomologies of Kac-Moody flag varieties*, Adv. Math. **197** (2005), no. 1, 198-221.

[HO] G. Heckman and E. Opdam, *Yang's system of particles and Hecke algebras*, Ann. Math. 2nd Ser., **145** no. 1 (1997), 139-173.

[HK] J. Hornbostel and V. Kiritchenko, *Schubert caculus for algebraic cobordism*, arxiv:0903.3936v3.

[Hu] T. Hudson, *Thom-Porteous formulas in algebraic cobordism*, arXiv:1206.2514

[Hu] S.G. Hulsurkar, *Proof of Verma's conjecture on Weyl's dimension polynomial*, Invent. Math. **27** (1974), 45–52.

[Ion03] B. Ion, *Involutions of double affine Hecke algebras*, Compositio Math. **139** (2003), no. 1, 67–84. MR2024965, arXiv:math.QA/0111010.

[Ion1] B. Ion, *Nonsymmetric Macdonald polynomials and Demazure characters*, Duke Math. J. **116** (2) (2003) 299-318, arXiv: math/0105061v2.

[Ion2] B. Ion, *A weight multiplicity formula for Demazure modules*, Int. Math. Res. Notices (2005) (5) (2005) 311-323.

[Ion3] B. Ion, *Nonsymmetric Macdonald polynomials and matrix coefficients for unramified principal series*, Adv. Math. **201** (2006) 36-62.

[Ion4] B. Ion, *Standard bases for affine parabolic modules and nonsymmetric Macdonald polynomials*, J. Algebra **319** (2008) 3480-3517.

[IS] B. Ion and S. Sahi, *Triple groups and Cherednik algebras*, American Math. Soc. ???????????, arXiv:math/0304186.

[Kac] V. Kac, *Infinite dimensional Lie algebras*, Third edition. Cambridge University Press, Cambridge, 1990. xxii+400 pp. ISBN: 0-521-37215-1; 0-521-46693-8, MR1104219.

[KP] V. Kac and D. Peterson, *Infinite dimensional Lie algebras, theta functions and modular forms*, Adv. in Math. **53** (1984), 125–264, MR0750341.

[KT95] M. Kashiwara, T. Tanisaki, *Kazhdan-Lusztig conjecture for affine Lie algebras with negative level.*, Duke Math. J. **77** (1995), no.1, 21–62.

[KT96] M. Kashiwara, T. Tanisaki, *Kazhdan-Lusztig conjecture for affine Lie algebras with negative level II: nonintegral case.*, Duke Math. J. **84** (1996), no.3, 21–62.

[KT98] M. Kashiwara, T. Tanisaki, *Kazhdan-Lusztig conjecture for symmetrizable Kac-Moody Lie algebras. III Positive rational case*, Asian J. Math. **2** (1998), no. 4, 779–832.

[KL] D. Kazhdan and G. Lusztig, *Proof of the Deligne-Langlands conjecture for Hecke algebras*, Invent. Math. **87** (1987), 153215, MR0862716.

[Kat85] S.-I. Kato, *On the Kazhdan-Lusztig polynomials for affine Weyl groups*, Advances in Math. **55** (1985) 103-130.

[KKR] S.V. Kerov, A.N. Kirillov, N. Yu. Reshetikhin, *Combinatorics, the Bethe ansatz and representations of the symmetric group*, J. Soviet Math. **41** (1988), no. 2, 916–924 MR0869576 (88i:82021)

[KR] A.N. Kirillov, N. Yu. Reshetikhin, *The Yangians, Bethe ansatz and combinatorics*, Lett. Math. Phys. textbf12 (1986), no. 3, 199–208. MR0865758 (88a:81181)

[KiKr] V. Kiritchenko and A. Krishna, *Equivariant cobordism of flag varieties and of symmetric varieties*, arXiv:1104.1089.

[KK] B. Kostant and S. Kumar, *T-equivariant K-theory of generalized flag manifolds*, Adv. in Math. **62** (1986), no. 3, 187-237.

[Ku] S. Kumar, *The nil Hecke ring and singularity of Schubert varieties*, Invent. Math. **123** (1996), 471-506, MR1383959.

[LM] M. Levine and F. Morel, *Algebraic Cobordism*, Springer Monographs in Mathematics, Springer, Berlin, 2007. xii+244 pp. ISBN: 978-3-540-36822-9; 3-540-36822-1 MR2286826.

[LSS] T. Lam, A. Schilling and M. Shimozono, *K-theory Schubert calculus of the affine Grassmannian*, Compos. Math. **146** (2010) 811852. MR2660675 (2011h:14078)

[LS] A. Lascoux and M.P. Schützenberger, *Décompositions dans l'algèbre des différences divisées*, Discrete Math. **99** (1992) 165-179, MR1158787.

[Lo] E. Looijenga, *Root systems and Elliptic curves*, Invent. Math. textbf38 (1976), 17-32.

[Lu] G. Lusztig, *Hecke algebras and Jantzen's generic decomposition patterns*, Adv. in Math. **37** (1980), no. 2, 121–164.

[Lu80] G. Lusztig, *Some problems in the representation theory of finite Chevalley groups*, The Santa Cruz Conference on Finite Groups (Univ. California, Santa Cruz, Calif., 1979), 313–317, Proc. Sympos. Pure Math., **37**, Amer. Math. Soc., Providence, R.I. (1980).

[Lu88] G. Lusztig, *Quantum deformations of certain simple modules over enveloping algebras*, Adv. in Math. **70** (1988), 237–249.

[Lu89] G. Lusztig, *Modular representations and quantum groups*, Contem. Math. **82** (1989), 58–77.

[Lu90] G. Lusztig, *On quantum groups*, J. of Algebra **131** (1990), 466–475.

[Lu94] G. Lusztig, *Monodromic systems on affine flag manifolds*, Proc. R. Soc. Lond. A **445** (1994), 231–246.

[Mac] I.G. Macdonald, *Lectures on Kac-Moody Lie algebras*, WHEN AND WHERE??

[Mac71] I.G. Macdonald, *Spherical functions on a group of p-adic type*, Publications of the Ramanujan Institute No. 2, Madras 1971.

[Mac72] I.G. Macdonald, *Affine root systems and Dedkind's η-function*, Invent. Math. **15** (1972) 91-143.

[Mac03] I.G. Macdonald, *Affine Hecke Algebras and Orthogonal Polynomials*, Cambridge Tracts in Mathematics, vol. **157**, Cambridge University Press, Cambridge, 2003. MR1976581.

[McN] P.J. McNamara, *Metaplecitc Whittaker functions and crystal bases*, Duke Math. J. **156** (2011) 1-31, arXiv: 0907.2675

[NS] P. Norbury and N. Scott, *Gromov-Witten invariants of $\mathbb{P}^1$ and Eynard-Orantin invariants*, Geometry and Topology **18** (2014) 1865-1910.

[N95] M. Noumi, *Macdonald-Koornwinder polynomials and affine Hecke rings*, Suriseisekikenkyusho Kokyuroku (FIX THE ACCENTS, see the references in [M03]) **919**(1995) 44–55 (in Japanese).

[Ok] S. Okada, *Applications of Minor Summation Formulas to Rectangular-Shaped Representations of Classical Groups*, J. Algebra **205** no. 2 (1998) 337-367.

[Ra] A. Ram, *Alcove walks, Hecke algebras, Spherical functions, crystals and column strict tableaux*, Pure and Applied Mathematics Quarterly **2** no. 4 (Special Issue: In honor of Robert MacPherson, Part 2 of 3) (2006), 963-1013.

[Ra] A. Ram, *Affine Hecke algebras and generalized standard Young tableaux*, J. Algebra **260** (2003) 367-415.

[RY] A. Ram and M. Yip, *A combinatorial formula for Macdonald polynomials*, arXiv:0803.1146

[S99] S. Sahi, *Nonsymmetric Koornwinder polynomials and duality*, Ann. Math. **150** (1999) no. 1, 267–282.

[ST] K. Saito and T. Takebayashi, *Extended affine rot systems III. Elliptic Weyl groups*, Publ. Res. Inst. Math. Sci. **33** (1997), no. 2, 301-329.

[Serre] J.-P. Serre, *Trees*, Springer-Verlag, 1980 (translated from the French original edition *Arbres, amagames et SL_2*, Astérisque No. 46, Soc. Math. France 1977, by John Stillwell).

[Slo] P. Slodowy, ??????????????

[Soe97] W. Soergel, *Kazhdan-Lusztig polynomials and a combinatoric for tilting modules*, Representation Theory 1 (1997), 83–114.

[Soe98] W. Soergel, *Character formulas for tilting modules over Kac-Moody algebras*, Representation Theory 2 (1998), 432–448.

[St] R. Steinberg, *Lectures on Chevalley groups*, Yale University 1967. CHECK THIS REFERENCE WITH MATHSCINET.

[St1975] R. Steinberg, *On a theorem of Pittie*, Topology **14** (1975), 173–177. MR0372897.

[To] B. Totaro, *The elliptic genus of a singular variety*, in *Elliptic cohomology*, 360364, London Math. Soc. Lecture Note Ser. **342** Cambridge Univ. Press, Cambridge, 2007, MR2330522

[vdL] H. van der Lek, *The homotopy type of complex hyperplane complements*, Ph.D. Thesis, Katholieke Universiteit Nijmegen, 1983.

[Wi1] M. Willems, *Cohomologie équivariante des tours des Bott et calcul de Schubert équivariant*, J. Inst. Math. Jussieu **5** (2006) 125–159, arXiv:0311079, MR2195948.

[Wi2] M. Willems, *K-théorie équivariante des tours des Bott. Application à la structure multiplicative de la K-théorie eq́uivariante des variétes des drapeaux*, Duke Math. J. **132** (2006) 271–309, arXiv:math.AG/0412152, MR2219259.

[Yip] M. Yip, *PhD thesis*, University of Wisconsin, Madison, 2010????????????