University of Melbourne, Working seminar A. Ram 07.12.2015

A ~~m/n~~ <u>parking function</u> $f:\{1,\ldots,n\}\to\mathbb{Z}_{\geq 0}$ is a function such that $(f(1),\ldots,f(n))$ arranged in ~~de~~increasing order,

$(a_1,a_2,\ldots,a_n)$ satisfies $a_i\leq i-1$.

<u>If $n=2$:</u> $(0,0)$ $(0,1)$, $(1,0)$ 3 total

<u>If $n=3$:</u> $(0,0,0)$ $(0,0,1)$ $(0,1,0)$ $(0,1,1)$ $(0,1,2)$

$(1,0,0)$ $(1,0,1)$ $(1,0,2)$

16 total

$(1,1,0)$ $(0,2,1)$

$(0,0,2)$ $(2,0,1)$

$(0,2,0)$ $(1,2,0)$

$(2,0,0)$ $(2,1,0)$

<u>A generalization:</u> $m=kn\pm 1$

A m/n parking function $f:\{1,\ldots,n\}\to\mathbb{Z}_{\geq 0}$ is a function such that $(f(1),\ldots,f(n))$ arranged in increasing order

$(a_1,a_2,\ldots,a_n)$ satisfies $a_i\leq mi+1$.

Hyperplane arrangements

For $1 \le i < j \le n$ let $H^{\varepsilon_i - \varepsilon_j} = \{(x_1, \ldots, x_n) \mid x_i - x_j = 0\}$.

and
$$\mathcal{A}^0 = \{H^{\varepsilon_i - \varepsilon_j} \mid 1 \le i < j \le n\}.$$

For $\alpha \in R^+$ let
$$H^\alpha = \{x \in \mathfrak{h}_{\mathbb{R}}^* \mid \alpha(x) = 0\} = \{x \in \mathfrak{h}_{\mathbb{R}}^* \mid s_\alpha x = x\}.$$

and
$$\mathcal{A}^0 = \{H^\alpha \mid \alpha \in R^+\}$$

Let
$$H^{-(\varepsilon_i - \varepsilon_j) + k\delta} = \{(x_1, \ldots, x_n) \mid x_i - x_j = k\}$$

and
$$\mathcal{A}^k = \{H^{-(\varepsilon_i - \varepsilon_j) + k\delta} \mid 1 \le i < j \le k\}$$

Let
$$H^{-\alpha + k\delta} = \{x \in \mathfrak{h}_{\mathbb{R}}^* \mid \alpha(x) = k\}$$

and
$$\mathcal{A}^k = \{H^{-\alpha + k\delta} \mid \alpha \in R^+\}$$

The <u>braid arrangement</u>, ~~or Coxet~~ is $\mathcal{A}^0$

The ~~Cat~~ <u>Shi arrangement</u> is $\mathcal{A}^0 \cup \mathcal{A}^1 = \mathcal{A}^{[0,1]}$

The <u>affine arrangement</u> is $\bigcup_{k \in \mathbb{Z}} \mathcal{A}^k = \mathcal{A}^{\mathbb{Z}}$.

The <u>k-Shi arrangement</u> is $\mathcal{A}^0 \cup \mathcal{A}^1 \cup \cdots \cup \mathcal{A}^k = \mathcal{A}^{[0,k]}$

The <u>Weyl group</u> is the set of conn. comps of $\mathbb{R}^n \setminus \mathcal{A}^0$

The <u>affine</u> Weyl gp is the set of conn. comps of $\mathbb{R}^n \setminus \mathcal{A}^{\mathbb{Z}}$

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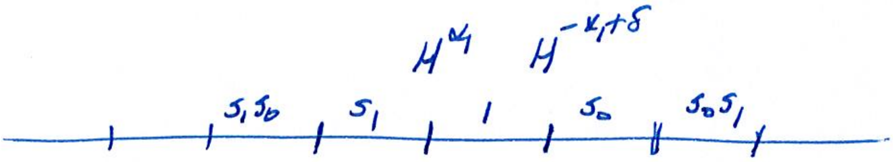
Type A_1: $\mathfrak{h}_{\mathbb{R}}^* = \mathbb{R}^1$ and $W_0 = \{1, s_1\}$

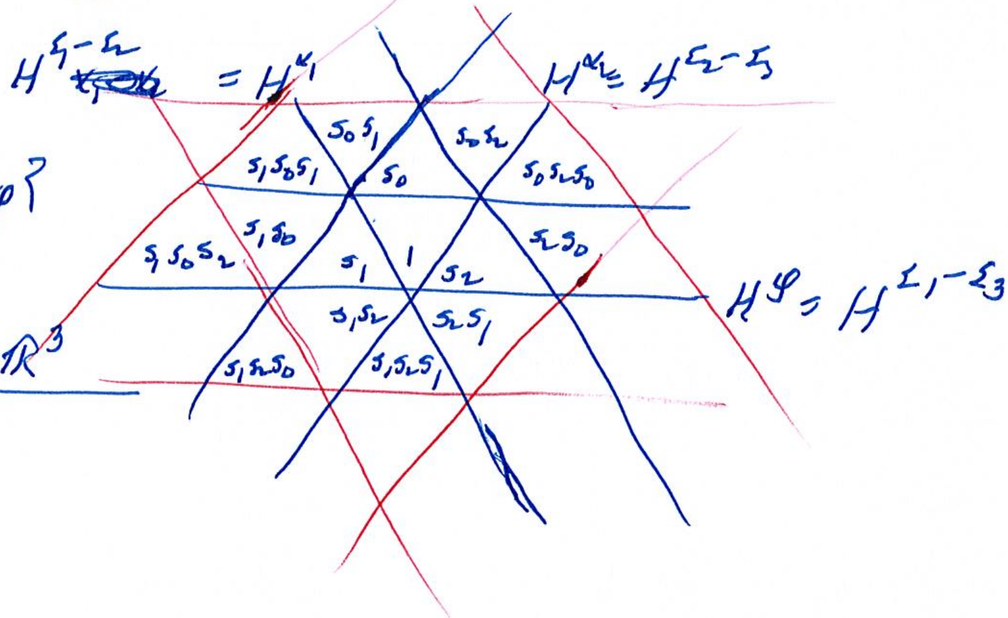


$W = \langle s_0, s_1 \mid s_0^2 = 1, s_1^2 = 1 \rangle$ an infinite dihedral group.

Type A_2: $\mathfrak{h}_{\mathbb{R}}^* = \mathbb{R}^2$ and $W_0 = \langle s_1, s_2 \mid s_1^2 = s_2^2 = 1, s_1 s_2 s_1 = s_2 s_1 s_2 \rangle$

$R^+ = \{\alpha_1, \alpha_2, \varphi\}$

$\mathfrak{h}_{\mathbb{R}}^* = \mathbb{R}^2 = \mathbb{R}^3$



$W = \langle s_0, s_1, s_2 \mid s_0^2 = s_1^2 = s_2^2 = 1, s_1 s_2 s_1 = s_2 s_1 s_2, s_0 s_1 s_0 = s_1 s_0 s_1, s_2 s_0 s_2 = s_0 s_2 s_0 \rangle$.

The map $\begin{array}{c} W \to W \\ w \mapsto w^{-1} \end{array}$ is a bijection.

The map $\left\{\begin{array}{c}\text{regions} \\ \text{of } \mathcal{A}^{\mathbb{Z}}\end{array}\right\} \longrightarrow \left\{\begin{array}{c}\text{regions} \\ \text{of } \mathcal{A}^{\mathbb{Z}}\end{array}\right\}$ is a bijection

$w \longmapsto w^{-1}$

(The restriction $\begin{array}{c} W_0 \to W_0 \\ w \mapsto w^{-1} \end{array}$ is a bijection i.e. $\left\{\begin{array}{c}\text{regions} \\ \text{of } \mathcal{A}^{0}\end{array}\right\} \to \left\{\begin{array}{c}\text{regions} \\ \text{of } \mathcal{A}^{0}\end{array}\right\}$)

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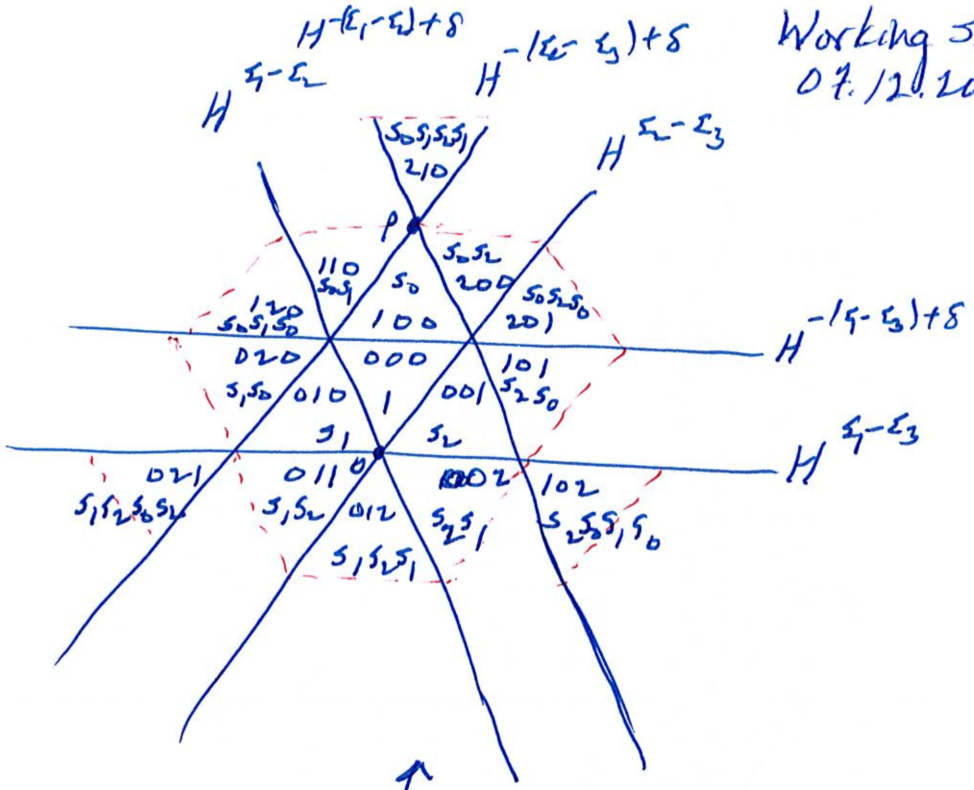
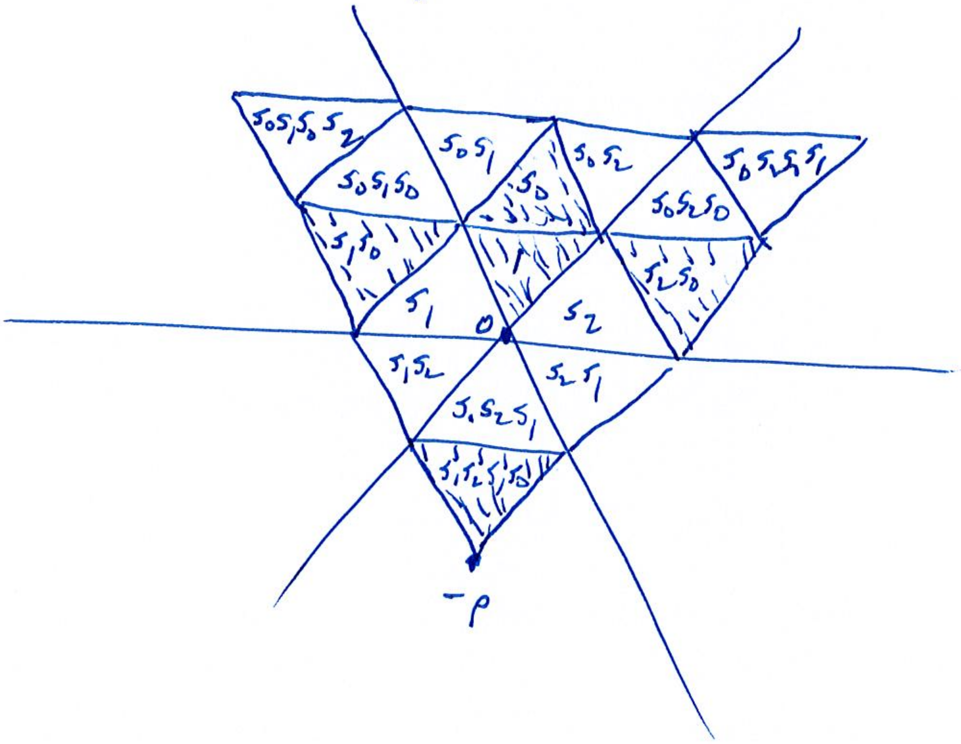
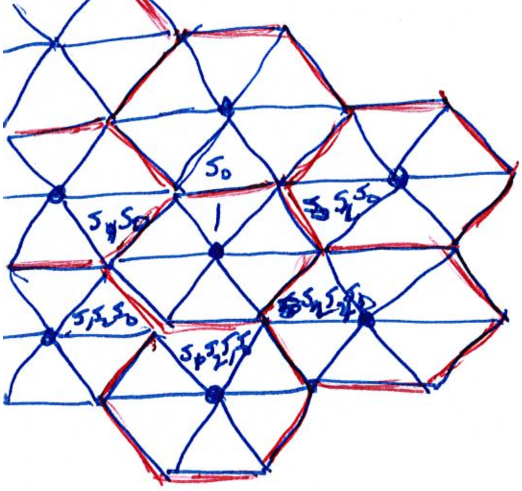
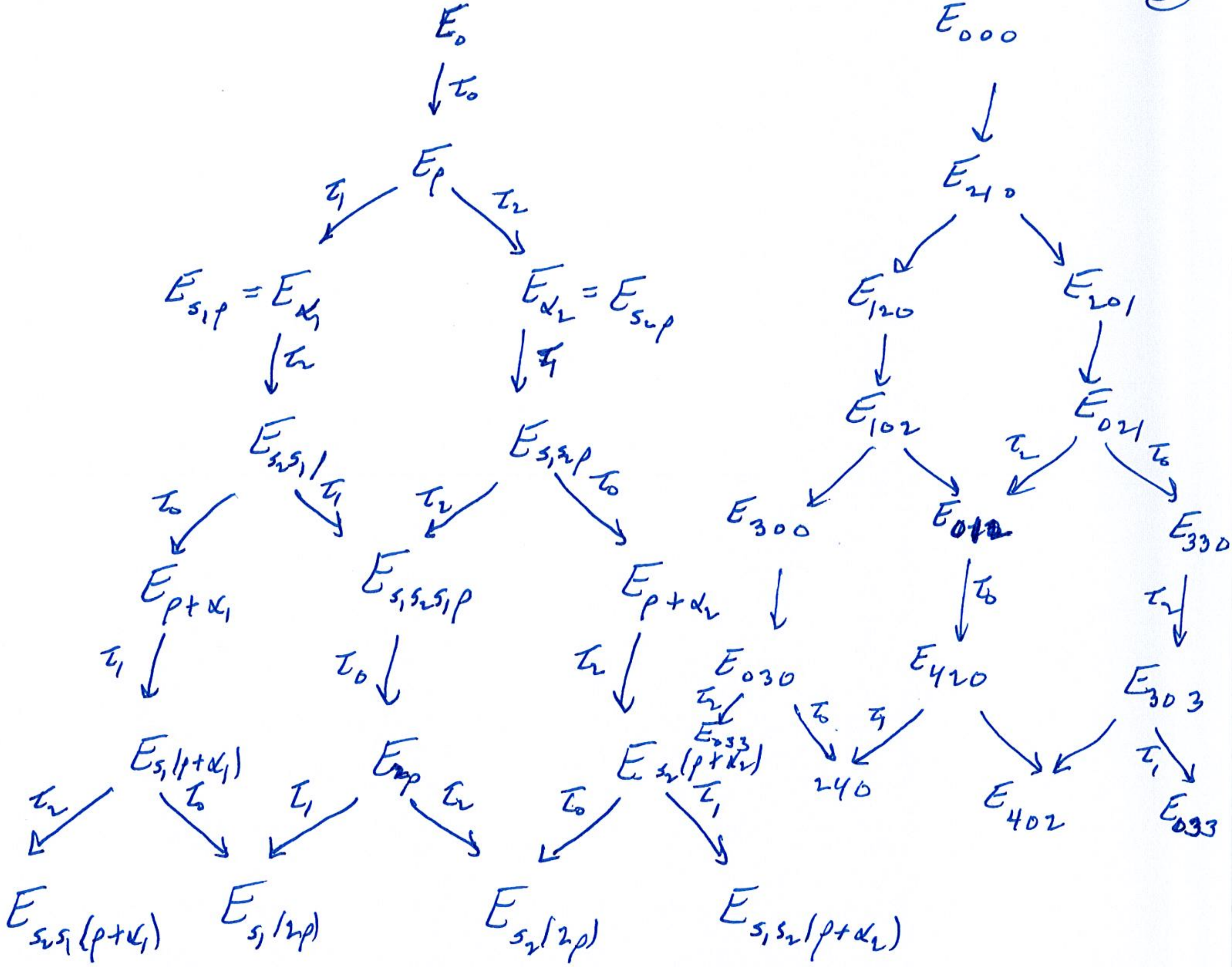


inverse





Irreducible DAHA modules



The double affine Hecke algebra $\tilde{H}$ is generated by q, $\tau_0, \dots, \tau_n$ and $y^{\omega_1}, \dots, y^{\omega_n}$ with relations

$y^\lambda \tau_i = y^{s_i\lambda}$ and $y^\lambda y^\mu = y^{\lambda+\mu}$, $\tau_i^2 =$

010 = 121, τ_1, τ_2, 100, 001

$\tau_i\tau_j\tau_i = \tau_j\tau_i\tau_j$ where $q = y^\delta$

Theorem The nonsymmetric Macdonald polynomials $\{E_w \mid w \in W_1\}$ form a basis of the irreducible finite dimensional DAHA module $L_{1+\frac{1}{h}}(triv)$.

Affine Springer fibers

$G = G(\mathbb{C}((t)))$

$K = G(\mathbb{C}[[t]])$

$$I = \left\{ \begin{pmatrix} \mathbb{C}[[t]]^\times & & \mathbb{C}[[t]] \\ & \ddots & \\ t\mathbb{C}[[t]] & & \mathbb{C}[[t]]^\times \end{pmatrix} \right\} \subseteq K.$$

The <u>affine flag variety</u> is G/I.

Let

$$N = \begin{pmatrix} 0 & & & & t \\ 1 & 0 & & & \\ & 1 & \ddots & & \\ & & & 0 & \\ & & & 1 & 0 \end{pmatrix}$$ ~~and~~

and the corresponding <u>affine Springer fiber</u> is

$$\mathcal{F}_{1+\frac{1}{n}} = \{ [illegible]\, gI \in G/I \mid NgI = gI \}$$

Define $\nu : \mathcal{F}_{1+\frac{1}{n}} \to W'$ by

Then

$$\mathcal{F}_{1+\frac{1}{n}} = \bigsqcup_{w \in W'} \nu^{-1}(w)$$ and ~~[illegible]~~

$$\nu^{-1}(w) \cong \mathbb{C}^{d_w} \quad \text{with} \quad d_w = \frac{n-1}{2} - \dim(w^{[illegible]}).$$