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Let $\nu = \frac{d}{m} \in \mathbb{Q}$ and $a \in \mathfrak{c}(F)^{rs}_\nu$.

Theorem 7.15 / Corollary 7.1.6

There is a graded action of

$$\mathcal{H}_\nu^{gr} \quad \text{on} \quad H^*_{G(\nu)}(Sp_a)$$

which commutes with the $\tilde{S}_a \times B_a$-action.

Prop. 8.2.2 and Theorem 8.2.3

C is the Chern filtration

There is a bigraded action of

$$\mathcal{H}_\nu^{rat} \quad \text{on} \quad Gr^C_* H^*_{\mathbb{G}_m(\nu)}(Sp_a)^{\tilde{S}_a \times B_a}$$

and, as $\mathcal{H}^{rat}_{\nu,\epsilon=1}$-modules

$$Gr^C_* H^*_{\mathbb{G}_m(\nu)}(Sp_a)^{\tilde{S}_a \times B_a} \cong L_\nu(triv)$$

the fin. dim. spherical $\mathcal{H}^{rat}_{\nu,\epsilon=1}$-module.

Here,

$$\tilde{S}_a \times B_a \overset{\text{see §5.4.6}}{=} \pi_1^{L_\nu}(\mathfrak{g}(F)^{rs}_\nu, K(a)) = \pi_1\left(\frac{\mathfrak{g}(F)^{rs}_\nu}{\tilde{L}_\nu}, K(a)\right)$$

with $\tilde{L}_\nu \overset{\text{see Lemma 3.3.5}}{=} G(F)^{\mathbb{G}_m(\nu)}$ and $F \overset{\text{see §2.0}}{=} \mathbb{C}((t))$.

Regular semisimple elements / conjugacy classes

$$\mathfrak{g} \underset{\chi}{\overset{\kappa}{\rightleftarrows}} \mathfrak{c}, \quad \text{where } \mathfrak{c} = \mathfrak{g}/\!/G \cong \mathfrak{t}/W_0.$$

κ is the <u>Kostant section</u> (see §2.3 and §2.3.2)

$$\mathfrak{c}(F)^{rs}_\nu = \{\text{homogeneous elts of slope } \nu\} = \left(\mathfrak{c}(F)^{rs}\right)^{\mathbb{G}_m(\nu)}$$
$$= \{a \in \mathfrak{c}(F)^{rs} \mid \text{if } s \in \mathbb{C}^\times \text{ then } s^d a(t) = a(s^m t)\}$$
$$\mathfrak{g}(F)^{rs}_\nu = \mathfrak{g}^{rs} \cap \chi^{-1}(\mathfrak{c}(F)^{rs}_\nu)$$

$$\mathfrak{g}(F)^{rs}_\nu \underset{\chi}{\overset{\kappa}{\rightleftarrows}} \mathfrak{c}(F)^{rs}_\nu$$

$$\kappa(a) = \gamma \longmapsto a = \chi(\gamma)$$

<u>Affine Springer fibers</u> (Defn. 5.2.1)

$$Sp_a \overset{\text{see } \S 5.3.2}{=} Sp_\gamma = \{gI \in G(F)/I \mid \mathrm{Ad}_{g^{-1}}(\gamma) \in \mathrm{Lie}(I)\}$$

$$= \{gI \in G(F)/I \mid g^{-1}\gamma g \in \mathrm{Lie}(I)\}$$

$$= \{gI \in G(F)/I \mid e^\gamma(gI) = gI\}$$

i.e. flags fixed by e^γ.

$\mathbb{G}_m^{cen}$, $\mathbb{G}_m^{rot}$, $\mathbb{G}_m^{dil}$ and $\mathbb{G}_m(\nu)$

The affine Kac-Moody group (see §2.5.2) is

$$G_{KM} = G^{cen} \rtimes \mathbb{G}_m^{rot} \quad \text{with} \quad 1 \to \mathbb{G}_m^{cen} \to G^{cen} \to G(F) \to 1$$

with torus

$$\mathbb{A}_{KM} = \mathbb{G}_m^{cen} \times \mathbb{A} \times \mathbb{G}_m^{rot},$$

and

$$X^*(\mathbb{G}_m^{rot}) = \mathbb{Z}\delta, \qquad X^*(\mathbb{G}_m^{cen}) = \mathbb{Z}\Lambda_{can} \qquad X^*(\mathbb{G}_m^{dil}) = \mathbb{Z}u.$$

$$X_*(\mathbb{G}_m^{rot}) = \mathbb{Z}\partial, \qquad X_*(\mathbb{G}_m^{cen}) = K_{can}$$

$\mathbb{G}_m^{dil}$ acts on $\mathfrak{g}(F)$ by (see §3.1.1)

$$\begin{aligned} \mathbb{G}_m^{dil} \times \mathfrak{g}(F) &\longrightarrow \mathfrak{g}(F) \\ (\lambda, x(t)) &\longmapsto \lambda x(t) \end{aligned}$$

$\mathbb{G}_m(\nu)$ is the one dimensional subtorus defined by (see §3.3.1)

$$\begin{aligned} \mathbb{G}_m(\nu) &\longrightarrow (G^{ad}(F) \rtimes \mathbb{G}_m^{rot}) \times \mathbb{G}_m^{dil} \\ s &\longmapsto (s^{d\rho^\vee}, s^m, s^{-d}) \end{aligned}$$

The graded Cherednik algebra

$$\mathcal{H}^{gr} = \mathbb{Q}[u,\delta] \otimes \mathbb{Q}[\Lambda_{can}] \otimes S(\mathfrak{a}^*) \otimes \mathbb{Q}[W]$$
$$= \mathbb{Q}[u] \otimes S(\mathfrak{h}^*) \otimes \mathbb{Q}[W]$$

with

(1) u is central

(2) $\mathbb{Q}[W]$ and $S(\mathfrak{h}^*)$ are subalgebras

(3) If $\lambda \in \mathfrak{h}^*$ then

$$t_{s_i} x_\lambda = x_{s_i\lambda} t_{s_i} + \langle \lambda, \alpha_i^\vee \rangle u, \quad \text{for } i \in \{0,1,\ldots,n\}$$
$$t_w x_\lambda = x_{w\lambda} t_w, \quad \text{for } w \in \Omega.$$

The rational Cherednik algebra

$$\mathcal{H}^{rat} = \mathbb{Q}[u,\delta] \otimes \mathbb{Q}[\Lambda_{can}] \otimes S(\mathfrak{a}^*) \otimes S(\mathfrak{a}) \otimes \mathbb{Q}[W_0]$$

with

(1) u and δ are central, Λ_0 commutes with t_w, $w \in W_0$.

(2) $S(\mathfrak{a}^*)$, $S(\mathfrak{a})$ and $\mathbb{Q}[W_0]$ are subalgebras

(3) If $\mu \in \mathfrak{a}$ and $\nu \in \mathfrak{a}^*$ then

$$t_w y_\mu = y_{w\mu} t_w, \quad t_w x_\nu = x_{w\nu} t_w \quad \text{and}$$

$$[y_\mu, x_\nu] = \langle \mu, \nu \rangle \delta + \tfrac{1}{2} u \sum_{\alpha \in R} c_\alpha \langle \nu, \alpha^\vee \rangle \langle \alpha, \mu \rangle t_{s_\alpha}$$

(4)
$$[y_{\mu^\vee}, \Lambda_{can}] = -x_\mu, \quad \text{where } \mathfrak{a} \to \mathfrak{a}^*$$
$$\mu^\vee \mapsto \mu = \langle \mu, \cdot \rangle$$

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For $\nu = \frac{d}{m} \in \mathbb{Q}$ define

$$\mathcal{H}_\nu^{gr} = \frac{\mathcal{H}^{gr}}{\langle u = -\frac{d}{m}\delta \rangle} \quad \text{and} \quad \mathcal{H}_\nu^{rat} = \frac{\mathcal{H}^{rat}}{\langle B_{KM} = 0,\ u = -\frac{d}{m}\delta \rangle}$$

where u, δ and

$$B_{KM} = \sum_{\alpha \in R} y_\alpha^2 + \delta \Lambda_{can} + \Lambda_{can}\delta \in \mathcal{H}^{rat}$$

are central elements.

These are the graded / rational Cherednik algebras

with central charge $\nu = \frac{+d}{m}$