Working seminar, Univ. Melbourne 04.04.2016 (0)

Hitchin fibers, Higgs bundles and Springer fibers

References:

(1) Section 6 of

A. Oblomkov and Z. Yun, Geometric representations of graded and rational Cherednik algebras, arXiv:1407.5685

(2) ~~Section~~ Lecture 4 of

Z. Yun, Lectures on Springer theories and orbital integrals, arXiv:1602.01451

The local to global morphism (Buchinger's hair).

The local to global morphism [OY (6.12)] is

family of affine Springer fibers over $\mathcal{A}_v^\heartsuit$

family of homogeneous Hitchin fibers

$$\beta_v : Sp_v \longrightarrow \mathcal{M}_v$$
$$gK \longmapsto (\mathcal{E}_0, \varphi_g)$$

where $K = G(\mathbb{C}[[t]])$

if $gK \in Sp_a$ (at $\mathcal{A}_v^\heartsuit$) then

$\mathcal{E}_0$ is the trivial K-torsor and

$\varphi_g = \mathrm{Ad}_{g^{-1}}(\kappa(a)) \in \mathcal{E}_0 \times^K \mathrm{Lie}(K)$.

Let P be a parahoric of $G(\mathbb{C}((t)))$.

The local to global morphism is

$$\beta_{P,v} : Sp_{P,v} \longrightarrow \mathcal{M}_{P,v}$$
$$gP \longmapsto (\mathcal{E}_0, \varphi_g)$$

where

if $gP \in Sp_{P,a}$ (at $\mathcal{A}_v^\heartsuit$) then

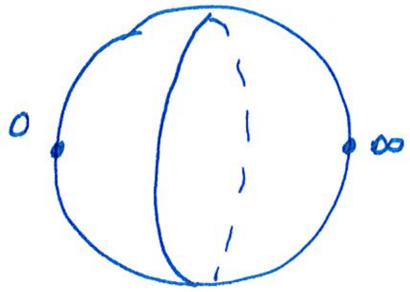
$\mathcal{E}_0$ is the trivial P-torsor and

$\varphi_g = \mathrm{Ad}_{g^{-1}}(\kappa(a)) \in \mathcal{E}_0 \times^P \mathrm{Lie}(P)$.

Principal G-bundles, Higgsfields and Hitchin moduli

Let X be an algebraic curve.

$X =$



The moduli stack of $\mathcal{G}_P$-torsors on X (princ. G-bundles), $\mathcal{G}$-torsors over X with P-level structure at 0, is

$$Bun_{\mathcal{G}_P} = \left\{ \mathcal{G}_P\text{-torsors } \begin{array}{c}\mathcal{E}\\ \downarrow\\ X\end{array} \right\}$$

Let $\begin{array}{c}\mathcal{L}\\ \downarrow\\ X\end{array}$ be a line bundle on X.

An $\mathcal{L}$-twisted $\mathcal{G}_P$-Higgs bundle (~~with~~ $\mathcal{L}$-twisted $\mathcal{G}$-Higgs bundle with P level structure at 0) is $(\mathcal{E}, \varphi)$ with the Higgs field

$\mathcal{E} \in Bun_{\mathcal{G}_P}$; φ a glopal section of $Ad_P(\mathcal{E}) \otimes \mathcal{L}$.

$$\begin{array}{c} Ad_P(\mathcal{E}) \otimes \mathcal{L} = (\mathcal{E} \times^{\mathcal{G}_P} Lie(\mathcal{G}_P)) \otimes \mathcal{L}. \\ \varphi \uparrow\downarrow \\ X \end{array}$$

The Hitchin moduli stack is

$$\mathcal{M}_{\mathcal{G}_P, \mathcal{L}} = \{ \mathcal{L}\text{-twisted } \mathcal{G}_P\text{-Higgs bundles } (\mathcal{E}, \varphi) \}$$

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The Hitchin fibration

Fix a line bundle $\begin{array}{c}\mathcal{L}\\ \downarrow \\ X\end{array}$ and let $\begin{array}{c}\mathfrak{c}_{\mathcal{L}} = \rho(\mathcal{L}) \times_X^{\mathbb{G}_m^{dil}} \mathfrak{c}_X .\\ \downarrow \\ X\end{array}$

conjugacy classes in $\mathfrak{g}$.

The <u>Hitchin base</u> is $\mathcal{A} = \{\text{sections of } \mathfrak{c}_{\mathcal{L}}\}$
The <u>Hitchin fibration</u> is

$$\begin{array}{ccc} f_P : \mathcal{M}_{P,\mathcal{L}} & \longrightarrow & \mathcal{A}_{\mathcal{L}} \\ \begin{array}{c}\mathrm{Ad}_P(E)\otimes\mathcal{L} \\ \varphi \circlearrowright \downarrow \\ X\end{array} & \longmapsto & (f_1(\varphi), \ldots, f_r(\varphi)) \end{array}$$

fundamental invariants in
$S(\mathfrak{g}^*)^G = S(\mathfrak{t}^*)^{W_0}$

The Hitchin base $\mathcal{A}$

$$\begin{Bmatrix}\text{homogeneous elts}\\ \text{of } \mathcal{A} \text{ of slope } \nu\end{Bmatrix} = \begin{array}{ccccl} \mathcal{A}_\nu & \subseteq & \mathcal{A} & & \\ \cup\!\shortmid & & \cup\!\shortmid & & \\ \mathcal{A}_\nu^\heartsuit & \subseteq & \mathcal{A}^\heartsuit & = & \text{gen reg semisimple locus of } \mathcal{A} \\ \cup\!\shortmid & & \cup\!\shortmid & & \\ \mathcal{A}_\nu^{ell} & \subseteq & \mathcal{A}^{ell} & = & \text{elliptic locus of } \mathcal{A} \end{array}$$

<u>[OY Prop. 6.3.7]</u> If $\deg \mathcal{L} \in \mathbb{Z}_{\geq 0}$ then

$\mathcal{M}_P^\heartsuit = \mathcal{M}_P\big|_{\mathcal{A}^\heartsuit}$ is a smooth Artin stack

$\mathcal{M}_P^{ell} = \mathcal{M}_P\big|_{\mathcal{A}^{ell}}$ is a Deligne-Mumford stack

$f_P^{ell} : \mathcal{M}_P\big|_{\mathcal{A}^{ell}} \longrightarrow \mathcal{A}^{ell}$ is proper.