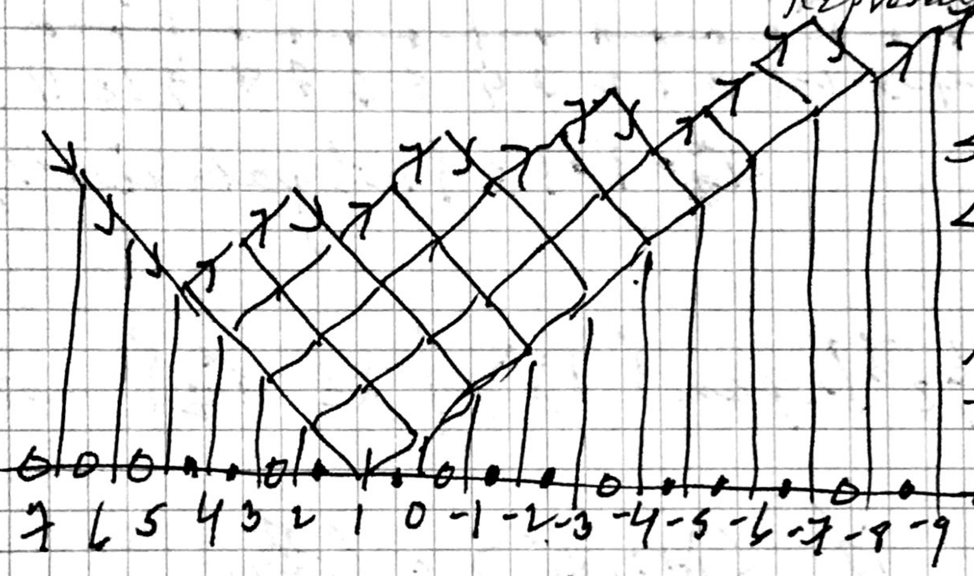
A FockSpace plan, Institut Henri Poincaré, A. Ram (1)

Type GL_∞ Partitions $\lambda = (4 4 3 3 2 2 1 1 1)$

conference on Asymptotic Representation Theory



Miwa-Jimbo-Date
Solitons and inf. dim Lie algebras

A. Ram
The Glass bead game

YouTube

$$|\lambda\rangle = v_4 \wedge v_3 \wedge v_1 \wedge v_0 \wedge v_{-2} \wedge v_{-3} \wedge v_{-5} \wedge v_{-6} \wedge v_{-7} \wedge v_{-9} \wedge v_{-10} \wedge \cdots$$

Kashiwara-Miwa-Stern 9508006

Leclerc-Thibon 9809122

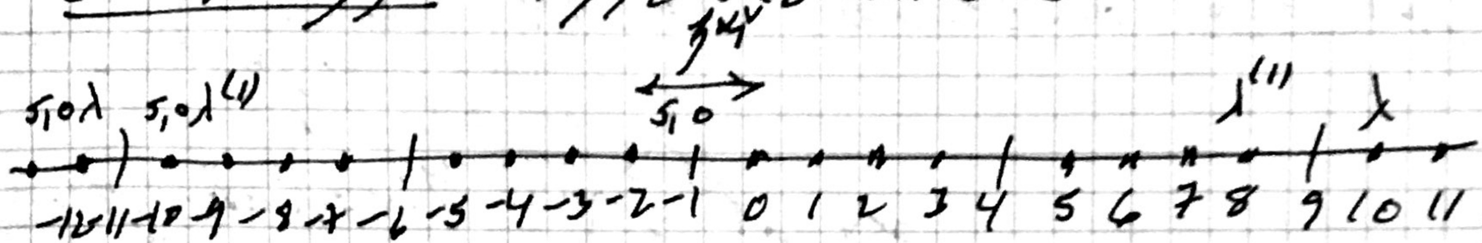
Let $\ell \in \mathbb{Z}_{>0}$.

$$F_\ell = \mathbb{Z}[t^{\frac{1}{2}}, t^{-\frac{1}{2}}]\text{-span}\left\{ v_{a_1} \wedge v_{a_2} \wedge \cdots \;\middle|\; \begin{array}{l} a_j \in \mathbb{Z} \\ a_j = -j+1 \text{ if } j >> 0 \end{array} \right\}$$

with

$$v_b \wedge v_a = \begin{cases} -(v_a \wedge v_b), & \text{if } a-b \in \ell\mathbb{Z}_{\geq 0} \\ -t^{\frac{1}{2}}(v_a \wedge v_b), & \text{if } 0 < a-b < \ell, \\ -t^{\frac{1}{2}}(v_{b-j} \wedge v_{a+j}) - (v_{a+j} \wedge v_{b-j}) - t^{\frac{1}{2}}(v_a \wedge v_b) & \\ \quad \text{if } a-b = k\ell + j \text{ with } k \in \mathbb{Z}_{>0} & \\ \quad \text{and } j \in \{1, 2, \ldots, \ell-1\}. & \end{cases}$$

General Type Type $\widehat{sl}_2$ with $\ell = 5$ (2)



$\mathfrak{a}^*_{\mathbb{Z}} = \mathbb{Z}\omega_1 = \{\text{integral weights}\}$.

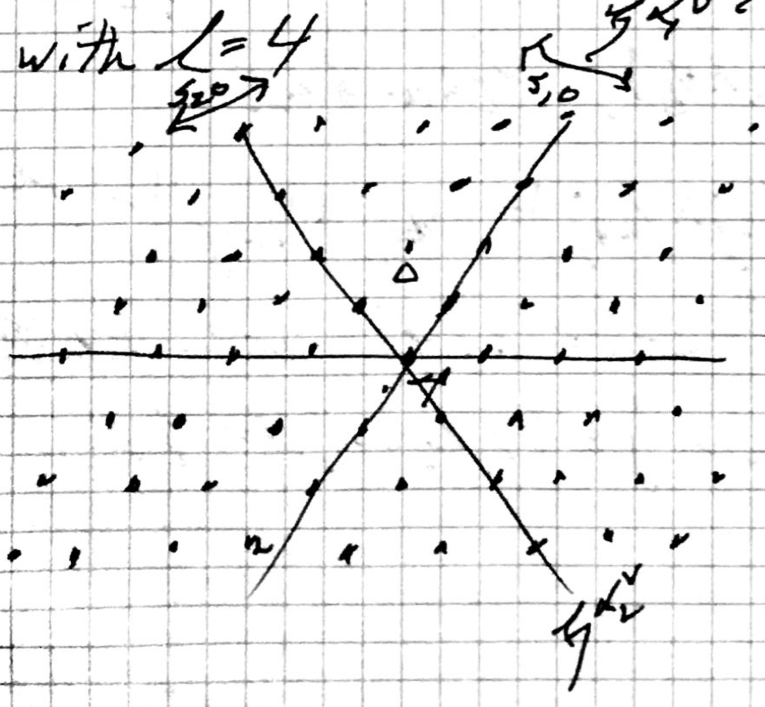
Let $\ell \in \mathbb{Z}_{>0}$.

$F_\ell = \mathbb{Z}[t^{\frac{1}{2}}, t^{-\frac{1}{2}}]\text{-span}\{ |\lambda\rangle \mid \lambda \in \mathfrak{a}^*_{\mathbb{Z}} \}$ with

$$|s_i \circ \lambda\rangle = \begin{cases} -|\lambda\rangle, & \text{if } \langle \lambda+\rho, \alpha_i^\vee\rangle \in \ell\mathbb{Z}_{\geq 0}, \\ -t^{\frac{1}{2}}|\lambda\rangle, & \text{if } 0 < \langle \lambda+\rho, \alpha_i^\vee\rangle < \ell, \\ -t^{\frac{1}{2}}|s_i \circ \lambda^{(1)}\rangle - |\lambda^{(1)}\rangle - t^{\frac{1}{2}}|\lambda\rangle, & \\ \quad \text{if } \langle \lambda+\rho, \alpha_i^\vee\rangle = k\ell + j \text{ with } k \in \mathbb{Z}_{\geq 0} & \\ \quad \text{and } j \in \{1, 2, \ldots, \ell-1\}. & \end{cases}$$

where $\lambda^{(1)} = \lambda - j\alpha_i$. $\langle \lambda+\rho, \alpha_i^\vee\rangle$ is distance from λ to $\mathfrak{h}^{\alpha_i^\vee}$.

Type $\widehat{sl}_3$, with $\ell = 4$



$W_0 = \langle s_1, s_2 \rangle$.

Bar involution $\overline{} : \mathcal{F}_\ell \to \mathcal{F}_\ell$ given by

$\overline{t^{\frac{1}{2}}} = t^{-\frac{1}{2}}$ and $\overline{|\lambda\rangle} = (\text{const})\, |w_0 \circ \lambda\rangle$

where w_0 is the longest element of

$W_0 = \langle s_1, s_2, \ldots, s_n \rangle$.

Two bases of $\mathcal{F}_\ell$:

$\{ |\lambda\rangle \mid \lambda \in (\mathfrak{a}_{\mathbb{Z}}^*)^+ \}$ and $\{ C_\lambda \mid \lambda \in (\mathfrak{a}_{\mathbb{Z}}^*)^+ \}$

with $\overline{C_\lambda} = C_\lambda$ and $C_\lambda = |\lambda\rangle + \sum_{\lambda \neq \mu} p_{\lambda\mu} |\mu\rangle$

with $p_{\lambda\mu} \in t^{\frac{1}{2}} \mathbb{Z}[t^{\frac{1}{2}}]$.

Main theorem Lanini-Ram-Sobaje 1612.03120

$p_{\lambda\mu} =$ singular parabolic ~~affine~~ KL polynomial for the affine Weyl group — decomposition numbers in representation theory.

Method of proof

$$\mathcal{F}_\ell \cong \bigoplus_{\nu \in A_{\ell-h}} \varepsilon_0 H 1_\nu$$

H ← affine Hecke algebra

1_ν ← symmetrizer

ε_0 ← antisymmetrizer.

Langlands to Whittaker

Lusztig Astérisque 101 1983

Nelsen-Ram 0401298

$Rep(G(\mathbb{C}))$

$\shortparallel$

$$\mathbb{C}[\mathfrak{a}_{\mathbb{Z}}^*]^{W_0} \xrightarrow{\sim} \mathbf{1}_0 H \mathbf{1}_0 \xrightarrow{\sim} \varepsilon_0 H \mathbf{1}_0$$

$$f \longmapsto f\mathbf{1}_0$$

$$h \longmapsto A_\rho h$$

Schur function s_λ $\longleftarrow$ $A_{\lambda+\rho} = \varepsilon_0 X^{\lambda+\rho} \mathbf{1}_0$

HL polynomial $P_\mu(0,t) \longleftarrow M_\mu = \mathbf{1}_0 X^\mu \mathbf{1}_0$

	Bosons	Fermions
Frenkel-Gaitsgory-Kazhdan-Vilonen 9703022	$C_c(K \backslash G/K)$	$C_c(N^- \backslash G/K)$
Chhaibi 13020932	Brownian motion	Whittaker

Macdonald version

DAHA = double affine Hecke algebra

$$\mathbb{C}[\mathfrak{a}_{\mathbb{Z}}^*]^{W_0} \xrightarrow{\sim} \mathbf{1}_0 \hat{H} \mathbf{1}_0 \xrightarrow{\sim} \varepsilon_0 \hat{H} \mathbf{1}$$

$$f \longmapsto f\mathbf{1}$$

$$h \longmapsto A_\rho h$$

$$P_\lambda(q, qt) \longleftarrow A_{\lambda+\rho} = \varepsilon_0 E_{\lambda+\rho}(q,t)\mathbf{1}$$

Macdonald polynomial $P_\mu(q,t) \longleftarrow M_\mu = \mathbf{1}_0 E_\mu(q,t)\mathbf{1}$