Fusion following Kazhdan-Lusztig:

Very different Lie algebras

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Fusion Talk. (1)
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$\hat{\mathfrak{g}} = \mathfrak{g}\otimes\mathbb{C}((\epsilon))\oplus\mathbb{C}K$ $\quad$ $V_1\otimes V_2$ is a $\hat{\mathfrak{g}}$-module.

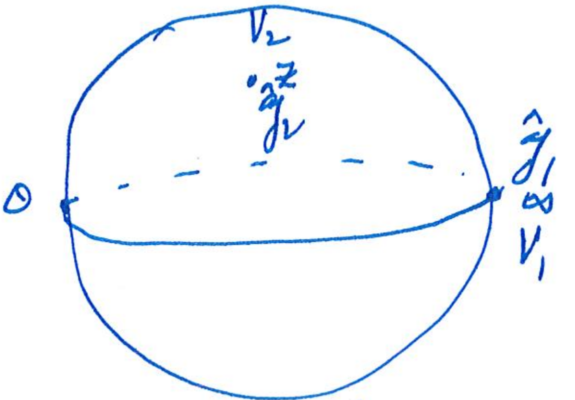
$\hat{\mathfrak{g}}_1 = \mathfrak{g}\otimes\mathbb{C}((\epsilon_1))\oplus\mathbb{C}K_1$ $\quad$ V_1 is a $\hat{\mathfrak{g}}_1$-module

$\hat{\mathfrak{g}}_2 = \mathfrak{g}\otimes\mathbb{C}((\epsilon_2))\oplus\mathbb{C}K_2$ $\quad$ V_2 is a $\hat{\mathfrak{g}}_2$-module

The arithmetic subgroup [SU, Prop 4.29]

M_C = moduli space of principal SL_2-bundles on C

$= SL_2(A_C)\backslash SL_2(F)/SL_2(\mathcal{O})$ = "$\Gamma\backslash G/K$".



$\epsilon_1 = \epsilon^{-1}$

$\epsilon_2 = \epsilon - z$

R = ring of regular functions on C.

$\mathfrak{F} = \mathfrak{g}\otimes R\oplus\mathbb{C}K_c$ acts on $V_1\otimes V_2$ by

$$(xf)(v_1\otimes v_2) = (xf^{(1)})v_1\otimes v_2 + v_1\otimes(xf^{(2)})v_2$$

for $x\in\mathfrak{g}$, $f\in R$, where

$f^{(1)}$ is expansion of f in ϵ_1

$f^{(2)}$ is expansion of f in ϵ_2

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Fusion $V_1 \boxtimes V_2$: Let

$\mathcal{R}_1 = \mathcal{R} \cap \epsilon\mathbb{C}[[\epsilon]]$ and $G_N = \left\{ (f_1 c_1) \cdots (f_N c_N) \,\middle|\, \begin{matrix} f_1, \ldots, f_N \in \mathcal{R}_1 \\ c_1, \ldots, c_N \in \mathfrak{g} \end{matrix} \right\}$

Then $V_1 \otimes V_2 \supseteq G_1(V_1 \otimes V_2) \supseteq G_2(V_1 \otimes V_2) \supseteq \cdots$ and

$$\frac{V_1 \otimes V_2}{G_1(V_1 \otimes V_2)} \leftarrow \frac{V_1 \otimes V_2}{G_2(V_1 \otimes V_2)} \leftarrow \cdots$$

$$V_1 \hat{\otimes} V_2 = \varprojlim \frac{V_1 \otimes V_2}{G_n(V_1 \otimes V_2)} = \left\{ (x_1, x_2, \ldots) \,\middle|\, \begin{matrix} x_i \in V_1 \otimes V_2 \\ x_{n+1} - x_n \in G_n(V_1 \otimes V_2) \end{matrix} \right\}$$

$\hat{\mathfrak{g}}$ acts on $V_1 \hat{\otimes} V_2$ by

$$(\omega c)(x_1, x_2, \ldots) = ((g_1 c) x_{q+1}, (g_2 c) x_{q+2}, \ldots)$$

where $q \in \mathbb{Z}_{\geq 0}$ and $g_1, g_2, \ldots \in \mathcal{R}$ with

$\omega \in \epsilon^{q}\mathbb{C}[[\epsilon]]$ and $g_n - \omega \in \epsilon^{n}\mathbb{C}[[\epsilon]]$.

Up to a dual, $V_1 \boxtimes V_2$ is

$$(V_1 \hat{\otimes} V_2)(\infty) = \left\{ x \in V_1 \hat{\otimes} V_2 \,\middle|\, \begin{matrix} \text{there exists } N \in \mathbb{Z}_{\geq 0} \text{ such that} \\ \text{if } c_1, \ldots, c_N \in \mathfrak{g} \text{ then} \\ (\epsilon c_1)(\epsilon c_2) \cdots (\epsilon c_N) x = 0 \end{matrix} \right\}$$