Niveaux de la forêt cristal, Colloquium University of Reims
A. Lascoux 21.03.2014 (1)

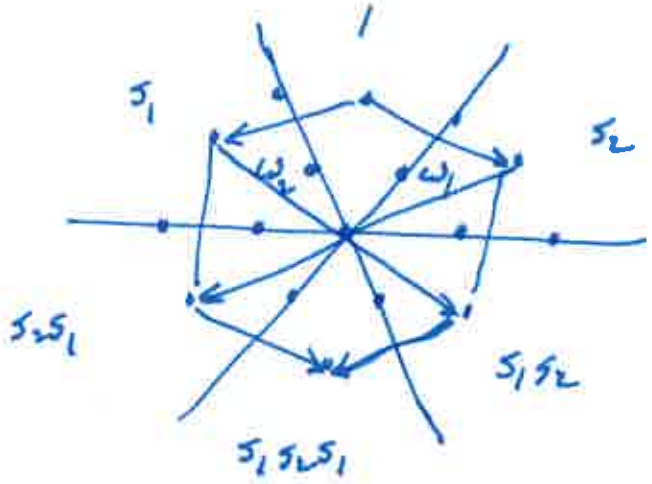
Mirrors

A_1

$s_1 = (-1)$

$W_0 = \{1, s_1\}$ with $s_1^2 = 1$ finite Weyl group.

A_2

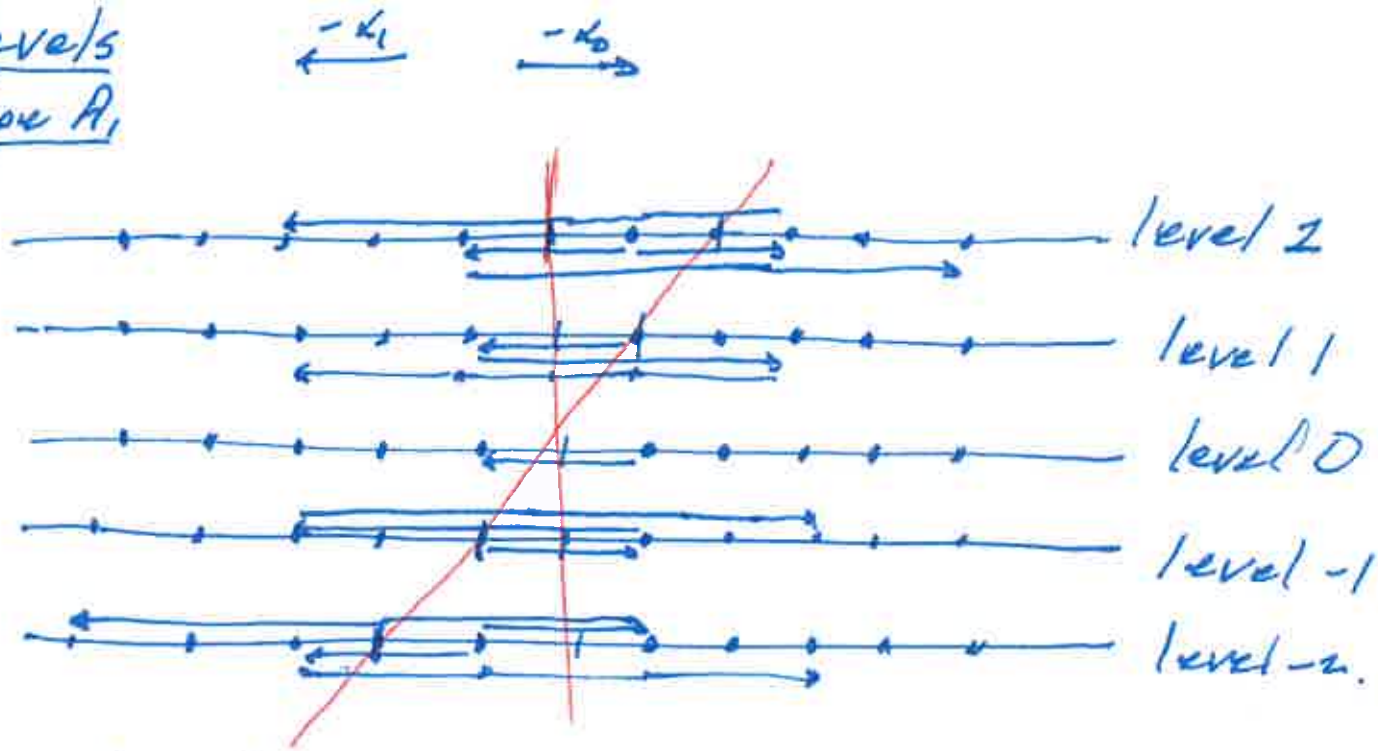


$$s_1 = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad s_2 = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

$W_0 = \{1, s_1, s_2, s_1 s_2, s_2 s_1, s_1 s_2 s_1\}$ with $s_1^2 = 1$, $s_2^2 = 1$, $s_1 s_2 s_1 = s_2 s_1 s_2$

Levels

Type A_1



$$s_1 = \left(\begin{array}{c|c} -1 & 0 \\ \hline 0 & 1 \end{array}\right) \qquad s_2 = \left(\begin{array}{c|c} -1 & 2 \\ \hline 0 & 1 \end{array}\right)$$

Reims Colloquium
21.03.2017
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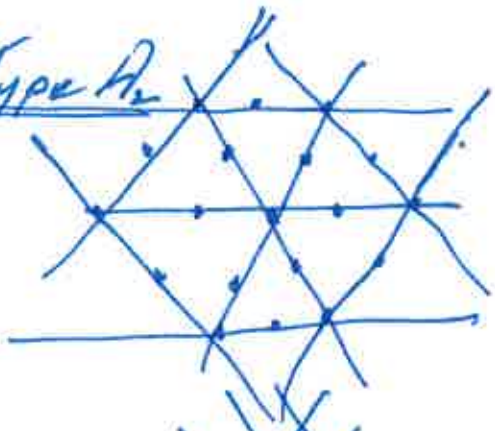
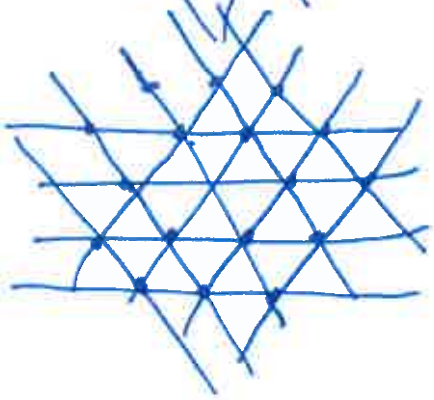
Levels: Type A_2

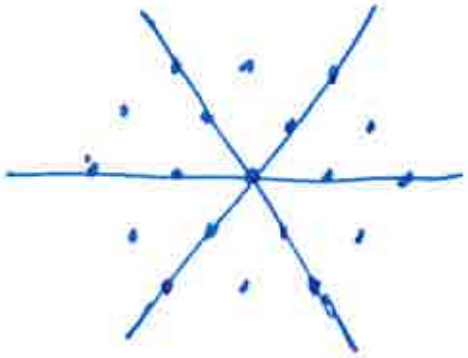
level 2



level 1



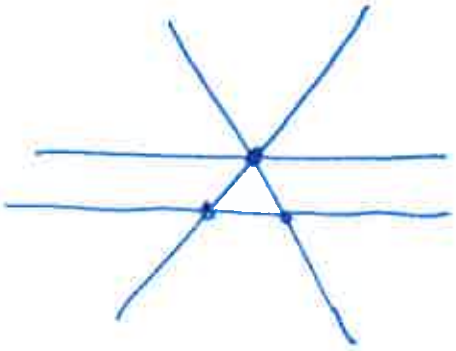
level 0



$$s_1 = \left(\begin{array}{cc|c} -1 & 0 & 0 \\ 1 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

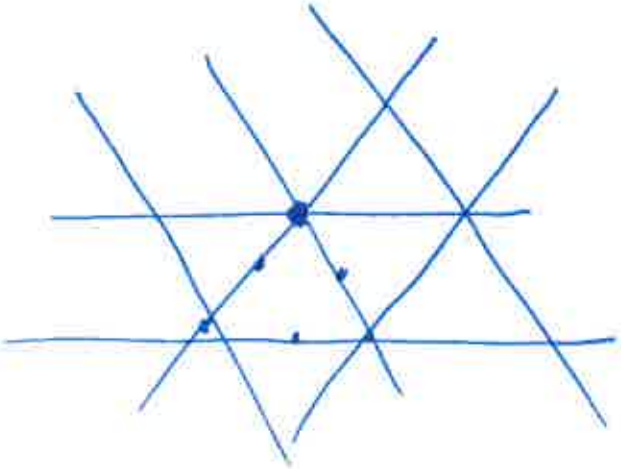
$$s_2 = \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

level -1

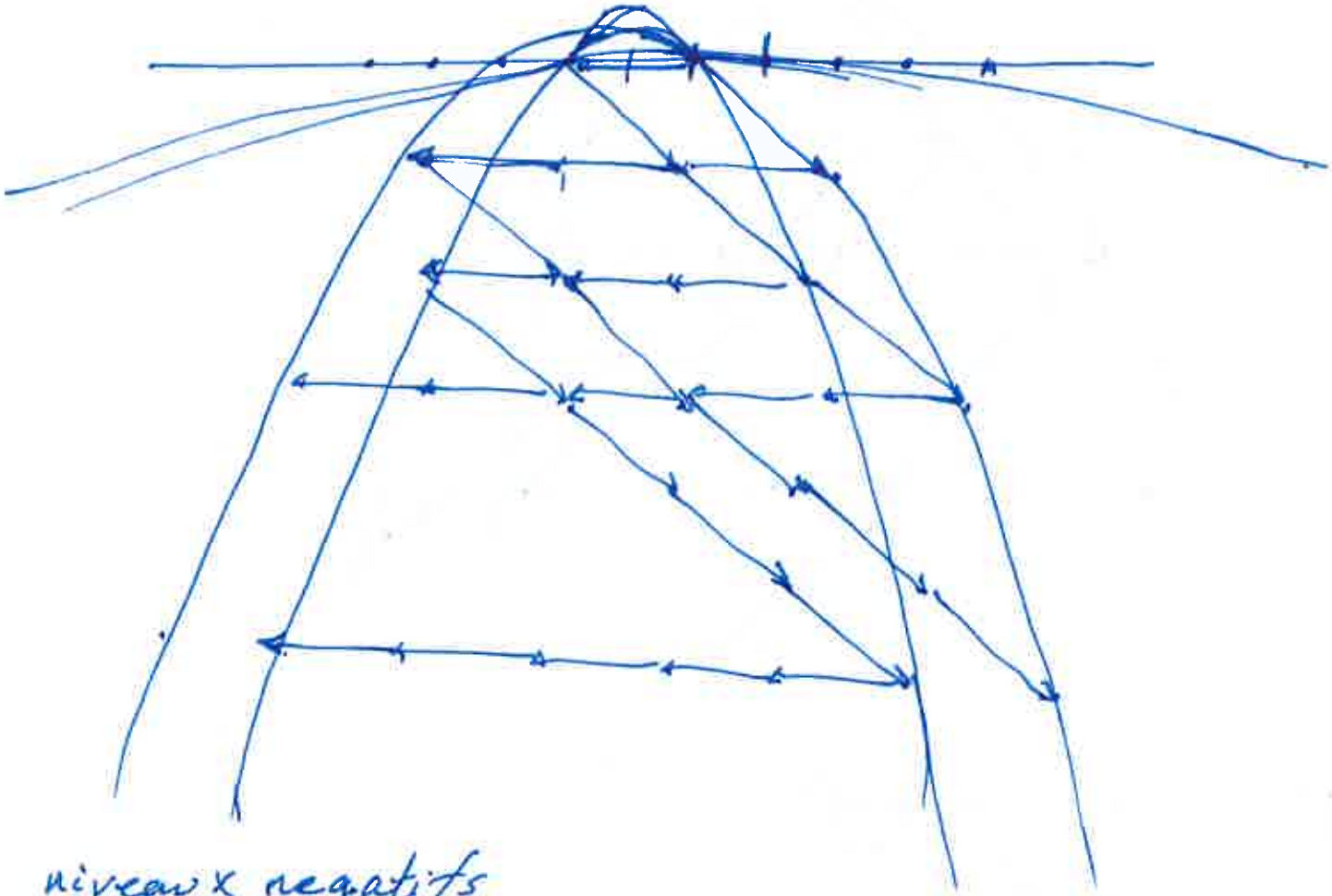


$$s_0 = \left(\begin{array}{cc|c} 0 & -1 & 1 \\ -1 & 0 & 1 \\ \hline 0 & 0 & 1 \end{array}\right)$$

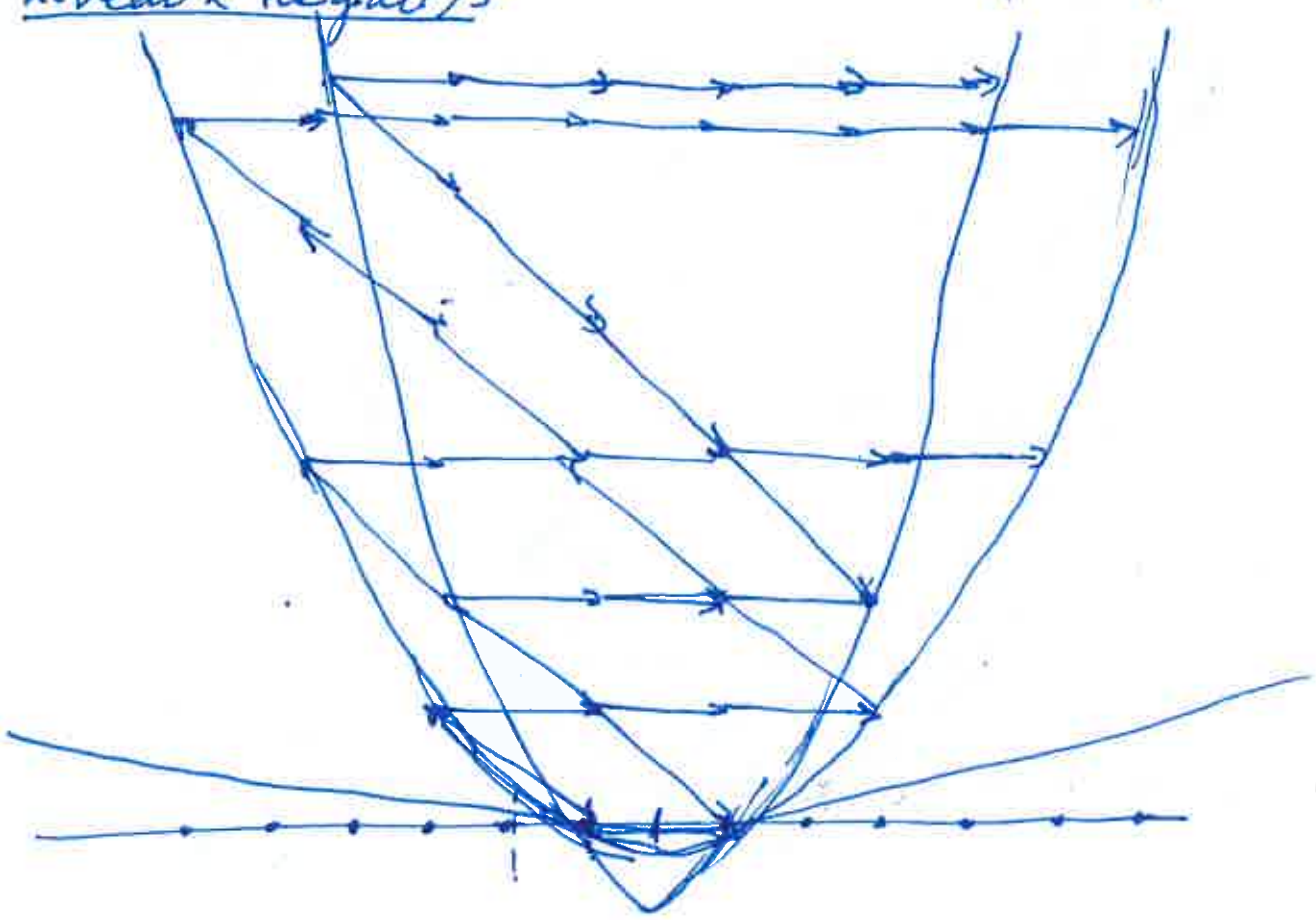
level -2



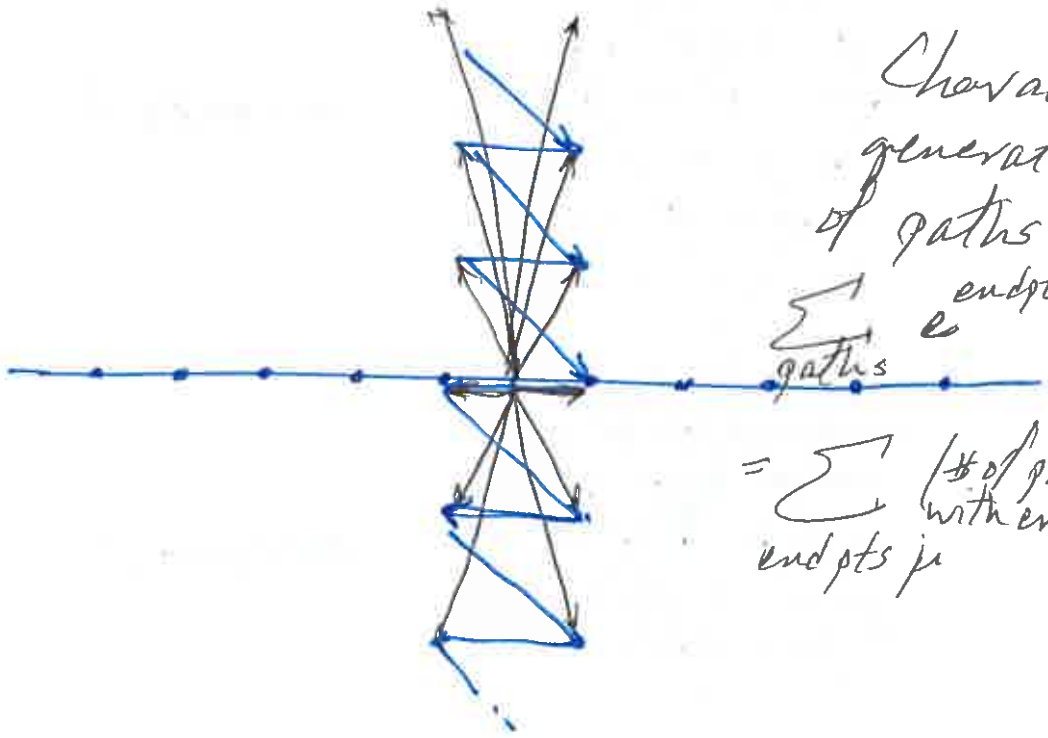
Cas A_1 : niveaux positifs



niveaux negatifs



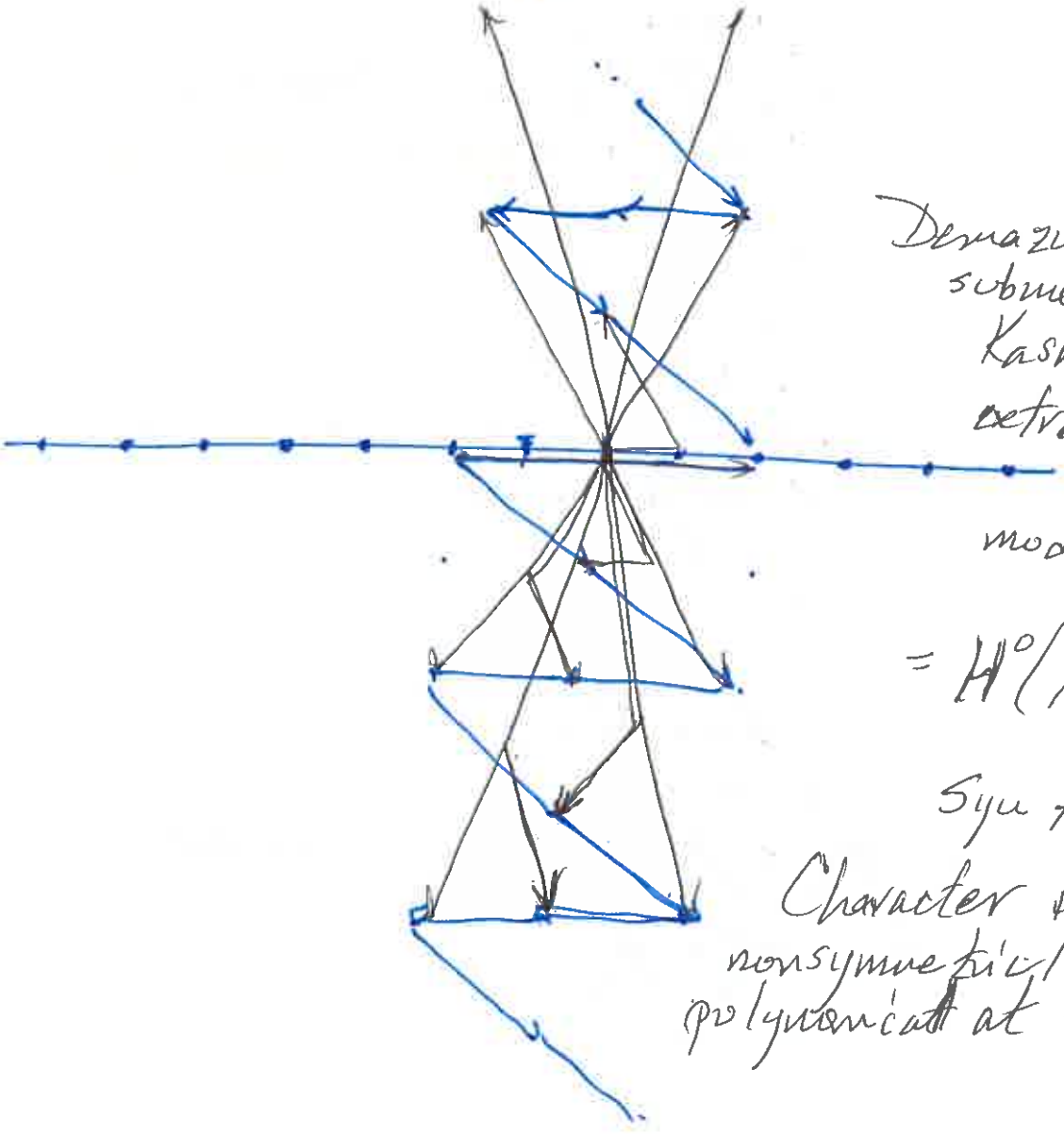
Cas A_1 : niveau 0



Character is generating function of paths

$$\sum_{\text{paths}} e^{\text{endpt}(p)}$$

$$= \sum_{\text{endpts } \mu} (\#\text{ of paths with endpt } \mu)\, e^{\mu}$$



Demazure submodule

Kashiwara's extremal weight module of level 0

$= H^0(X_1, \mathcal{L}_{w_1})$

Syu Kato

Character is a nonsymmetric Macdonald polynomial at $t = \infty$.

Chemins de Littelmann

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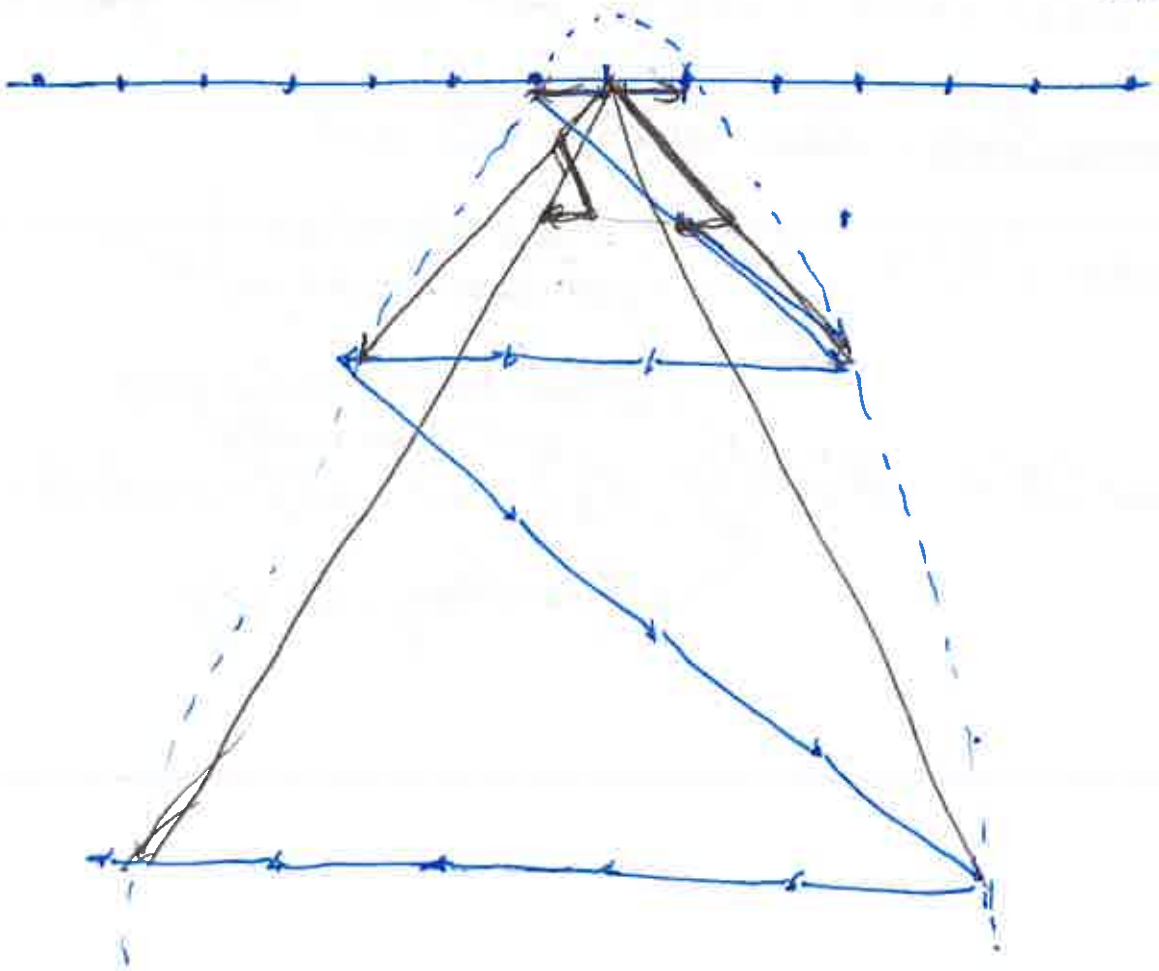

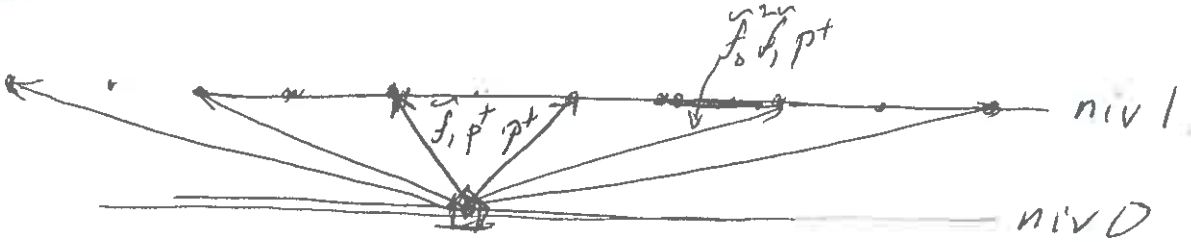

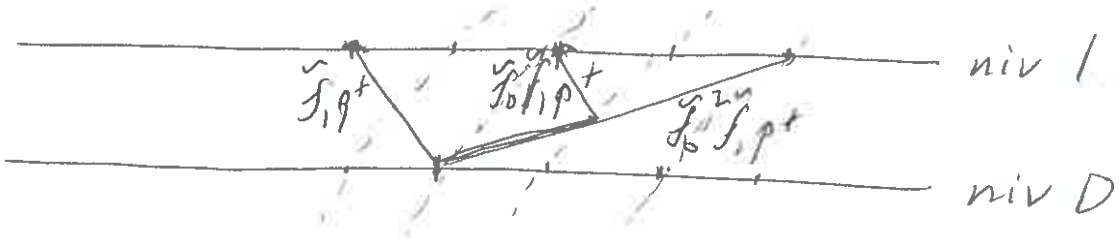

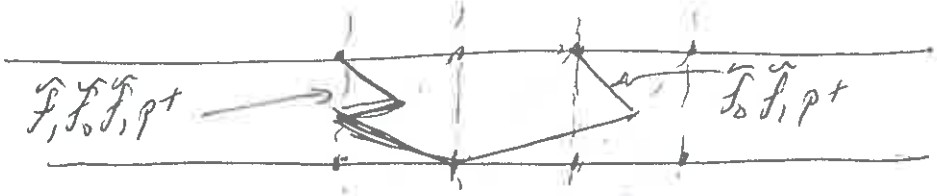

A crystal is a set B of paths which is closed under the operations $\tilde{f}_0$ and $\tilde{f}_1$.

The character of a crystal B is

$$\mathrm{char}(B) = \sum_{p \in B} e^{\mathrm{end}(p)}.$$

Kato and Borel-Weil-Bott

$$\mathrm{char}(H^i(G/I_+, \mathcal{L}_\lambda)) = \mathrm{char}(B(\lambda))$$

$$\mathrm{char}(H^i(X_w, \mathcal{L}_\lambda)) = \mathrm{char}(B(\lambda)_{\geq w})$$

$$\mathrm{char}(H^i((G/I_0)_{\leq w}, \mathcal{L}_\lambda)) = \mathrm{char}(B^{\mathrm{rev}}(\lambda)_{\geq w})$$