Pure math seminar
Prelim talk
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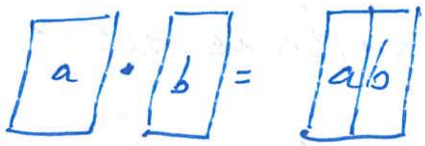
Preliminary talk

The symmetric group

$S_3 = \{$  $\}$

with product  acts on

$$V = \mathrm{span}\{v_{123}, v_{213}, v_{132}, v_{321}, v_{312}, v_{231}\}.$$

For example,

 $\cdot\,(5v_{123} + 7v_{321} + \sqrt{10}\, v_{231}) = 5v_{321} + 7v_{123} + \sqrt{10}\, v_{213}.$

In matrix form

$M_1 =$  $= \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$

$M_2 =$  $= \begin{pmatrix} 0&1&0&0&0&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&1&0\\ 0&0&0&1&0&0\\ 0&0&1&0&0&0 \end{pmatrix}$

 $= \begin{pmatrix} 0&0&0&1&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&1&0\\ 1&0&0&0&0&0\\ 0&0&1&0&0&0\\ 0&1&0&0&0&0 \end{pmatrix}$

 $= \begin{pmatrix} 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&1&0&0&0&0\\ 0&0&0&1&0&0 \end{pmatrix}$

$$M_3 = \overline{\times} + \ast = \begin{pmatrix} 0&0&1&1&0&0 \\ 0&0&0&0&1&1 \\ 1&0&0&0&1&0 \\ 1&0&0&0&0&1 \\ 0&1&1&0&0&0 \\ 0&1&0&1&0&0 \end{pmatrix}$$

Amazing theorem

M_1, M_2, M_3 commute.

Can you prove it?

Problem Find simultaneous eigenvectors

$e_1, e_2, e_3, e_4, e_5, e_6$ for M_1, M_2, M_3

Example Let $e_1 = v_1 + v_2 + v_3 + v_4 + v_5 + v_6$

Then

$M_1 e_1 = e_1, \quad M_2 e_2 = e_2$ and $M_3 e_3 = 2e_3$

Example Let $e_6 = v_1 - v_2 - v_3 - v_4 + v_5 + v_6$

Then

$M_1 e_6 = e_6, \quad M_2 e_6 = -e_6, \quad M_3 e_6 = -2e_6.$