27.05.2024
GA Talk
A. Ram

①

Macdonald elements

Integral form Macdonald elements

$$J_\mu(q,t) = \sum_\nu \mathrm{Card}\left(Y^{-1}_{I s_\nu I}(u_\mu)\right) k_\nu$$

Modified Macdonald elements

$$\tilde{H}_\mu(q,t) = \sum_\pi \mathrm{Card}\left(Y^{-1}_{I_\pi}(u_\mu)\right) 1_\pi .$$

Loop Groups

$$\begin{array}{lcl}
\tilde{G} = GL_n(\mathbb{F}_q((\epsilon))) & & \\
\cup\!| & & \\
K = GL_n(\mathbb{F}_q[[\epsilon]]) & \xrightarrow{\epsilon = 0} & G = GL_n(\mathbb{F}_q) \\
\cup\!| & & \cup\!| \\
I_\pi & \longrightarrow & P_\pi = \left\{ \begin{pmatrix} \ddots & * \\ 0 & \ddots \end{pmatrix} \right\} \\
 & & \text{block sizes } (\pi_1, \ldots, \pi_\ell) \\
\cup\!| & & \cup\!| \\
I & \longrightarrow & B = \left\{ \begin{pmatrix} * & & * \\ & \ddots & \\ 0 & & * \end{pmatrix} \right\}
\end{array}$$

$$G = \bigsqcup_{w \in W} BwB \quad \text{and} \quad P_\pi = \bigsqcup_{w \in W_\pi} BwB$$

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Notation

$\mathcal{W}$ is an index set for the conjugacy classes of W

γ_ν is a minimal length element in the conjugacy class W_ν of W.

$\mathcal{G}^{unip}$ is an index set for unipotent conjugacy classes in G

$$u_\mu = \begin{pmatrix} \boxed{\begin{matrix} 1 & 1 & 0 \\ & \ddots & 1 \\ 0 & & 1 \end{matrix}} & & \\ & \ddots & \\ & & \boxed{\begin{matrix} 1 & 1 & 0 \\ & \ddots & 1 \\ 0 & & 1 \end{matrix}} \end{pmatrix}$$

u_μ is a representative of conjugacy class of G indexed by μ.

Affine Lusztig varieties

$$Y^{-1}_{I\gamma_\nu I}(u_\mu) = \{ gI \in G/I \mid g^{-1}u_\mu g \in I\gamma_\nu I \}$$

π-parabolic affine Springer fibers

$$Y^{-1}_{I_\pi}(u_\mu) = \{ gI_\pi \in G/I_\pi \mid g^{-1}u_\mu g \in I_\pi \}$$

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Hecke algebra

$$\mathbb{1}_B^G = \mathrm{Ind}_B^G(\mathrm{triv})$$

The Iwahori-Hecke algebra is

$$H = \mathrm{End}_G(\mathbb{1}_B^G) \text{ with basis } \{T_w \mid w \in W\}$$

so that

$$\lim_{q\to 1} H = \mathbb{C}W \quad \text{and} \quad \lim_{q\to 1} T_w = w.$$

As $\mathbb{C}$-algebras $H \cong \mathbb{C}W$.

The π-parabolic projector is

$$\mathbb{1}_\pi = \sum_{w\in W} c_{w\pi} T_w, \quad \text{where}$$

$$c_{w\pi} = \begin{cases} 0, & \text{if } w \notin W_\pi, \\ 1, & \text{if } w \in W_\pi. \end{cases}$$

The matrix $C = (c_{w\pi})$ with $w \in W$, $\pi \in P$ is the contraction matrix

$$\mathrm{Card}(Y_{B\dot{\gamma}_v B}^{-1}(u_\mu)) = \mathrm{Tr}(u_\mu T_{\gamma_v}, \mathbb{1}_B^G)$$

$$\mathrm{Card}(Y_{P_\pi}^{-1}(u_\mu)) = \mathrm{Tr}(u_\mu, \mathbb{1}_B^G \mathbb{1}_\pi)$$

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The Geck-Rouquier basis of $Z(H)$ is

$\{K_\nu \mid \nu \in \widehat{W}\}$, where $K_\nu = \sum_{w \in W} \kappa_{\nu w}(q) T_w$.

such that

$\kappa_{\nu, \gamma_\mu}(q) = \delta_{\mu\nu}$, if γ_μ is minimal length in $\mathcal{W}_\mu$,

$$\kappa_{\nu, w}(1) = \begin{cases} 0, & \text{if } w \notin \mathcal{W}_\nu, \\ 1, & \text{if } w \in \mathcal{W}_\nu. \end{cases}$$

For example, if $G = GL_3(\mathbb{F}_q)$ then

$$K = (\kappa_{\nu w}) = \begin{array}{c} (1^3) \\ (21) \\ (3) \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & q & q-1 & q-1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

and

$$C = (c_{w\pi}) = \begin{array}{ccc} (1^3) & (21) & (3) \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

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Symmetric functions (type GL_n only)

Integral form Macdonald polynomials

$$J_\mu(x;q,t) = \sum_\nu \mathrm{Card}\left(\mathcal{Y}^{-1}_{I\gamma_\nu I}(u_\mu)\right) m_\nu$$

Modified Macdonald polynomials

$$\tilde{H}_\mu[X;q,t] = \sum_\pi \mathrm{Card}\left(\mathcal{Y}^{-1}_{\mathcal{L}_\pi}(u_\mu)\right) m_\pi$$

where m_π denotes monomial symmetric functions.

Plethystic Transformation

$$\tilde{H}_\mu[X;q,t] = J_\mu\left[\frac{X}{1-t^{-1}};q,t^{-1}\right] t^{n(\mu)}$$

Let $R_{\nu\pi}(q)$ be given by

$$m_\nu\left[\frac{X}{1-q^{-1}}\right] = \sum_\pi R_{\nu\pi}(q)\, m_\pi$$

Then

$$R_{\nu\pi}(q) = \sum_{w\in S_n} q^{n-\ell(w)} K_{\nu w}(q)\, C_{w\pi} \frac{1}{\mathrm{Card}(P_\pi/B)}$$