Discoverers of Murphy elements:

Murphy 1981

Jucys 1966, 1971

Cherednik 1989

Dipper-James 1988

Hoefsmit 1974

Reshetikhin 1987

Ariki-Koike 1994

Young ??

Nazarov 1996

A. Ram, "Seminormal representations of Weyl groups and Iwahori-Hecke algebras" to appear in Proc. London Math. Soc. 1997

A_n, B_n, D_n, G_2

A. Ram and D.E. Taylor, "Explicit irreducible representations of the Iwahori-Hecke algebra of type F_4", preprint.

F_4

R. Leduc and A. Ram,
"A Ribbon Hopf algebra approach to the irreducible representations of centralizer algebras"
Adv. Math 1997

Some BW rep.

$M_k =$

$M_\ell =$

JUCYS - MURPHY ELEMENTS

Theorem

(1) The M_k generate a commutative subalgebra

(2) Every irreducible representation has a unique basis of simultaneous eigenvectors

$$M_k v_L = CT(L(k)) v_L$$

(3) The action is given by

$$T_i v_L = (T_i)_{LL} v_L + \left(q^{-1} + (T_i)_{LL}\right) v_{s_i L}$$

where $$(T_i)_{LL} = \frac{q - q^{-1}}{1 - q^{CT(L(i-1)) - CT(L(i))}}$$

(4) $$\frac{dM_k}{dq} = \left.\frac{M_k^2 - 1}{q - q^{-1}}\right|_{q=1} = m_k$$

(5) $$M_k = T_{w_{0,k}} T_{w_{0,k-1}}^{-1}$$

Where $w_{0,k}$ is the longest element in

$w_{0,k-1}$ is the longest element in

<u>Iwahori-Hecke algebra Type A_{n-1}</u>:

$s_i = (i, i-1)$

$$M_k = T_k T_{k-1} \cdots T_2 T_2 \cdots T_k$$

$$CT(L(k)) = q^{2c(L(k))}$$

L a standard tableaux, $L(k) =$ box containing k, $c(x) = j - i$.

<u>Symmetric group</u> $\mathbb{C}S_n$

$$m_k = \sum_{i=2}^{k} (i-1, k)$$

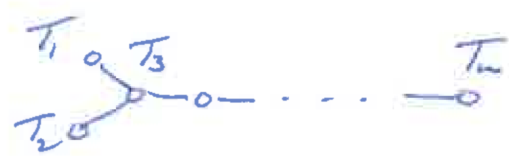
$$CT(L(k)) = c(L(k)).$$

<u>Iwahori-Hecke algebra type B_n</u>:

$$M_k = T_k T_{k-1} \cdots T_2 T_1 T_2 \cdots T_k.$$

$$CT(L(k)) = \mathrm{sgn}(L(k))\, p^{\mathrm{sgn}(L(k))} q^{2ct(L(k))}$$

<u>Iwahori-Hecke algebra type D_n</u>:



$$M_k = T_k \cdots T_3 T_2 T_1 T_3 T_4 \cdots T_k.$$

$$CT(L(k)) = \mathrm{sgn}(L(1))\,\mathrm{sgn}(L(k))\, q^{2ct(L(k))}.$$

Discoverers of Murphy elements:

Murphy 1981

Jucys 1966, 1971

Cherednik 1989

Hoefsmit 1974

Reshetikhin 1987

Ariki-Koike 1994

Nazarov 1996

Young ??

A. Ram, "Seminormal representations of Weyl groups and Iwahori-Hecke algebras"
to appear in Proc. London Math Soc. 1997

A_n, B_n, D_n, G_2

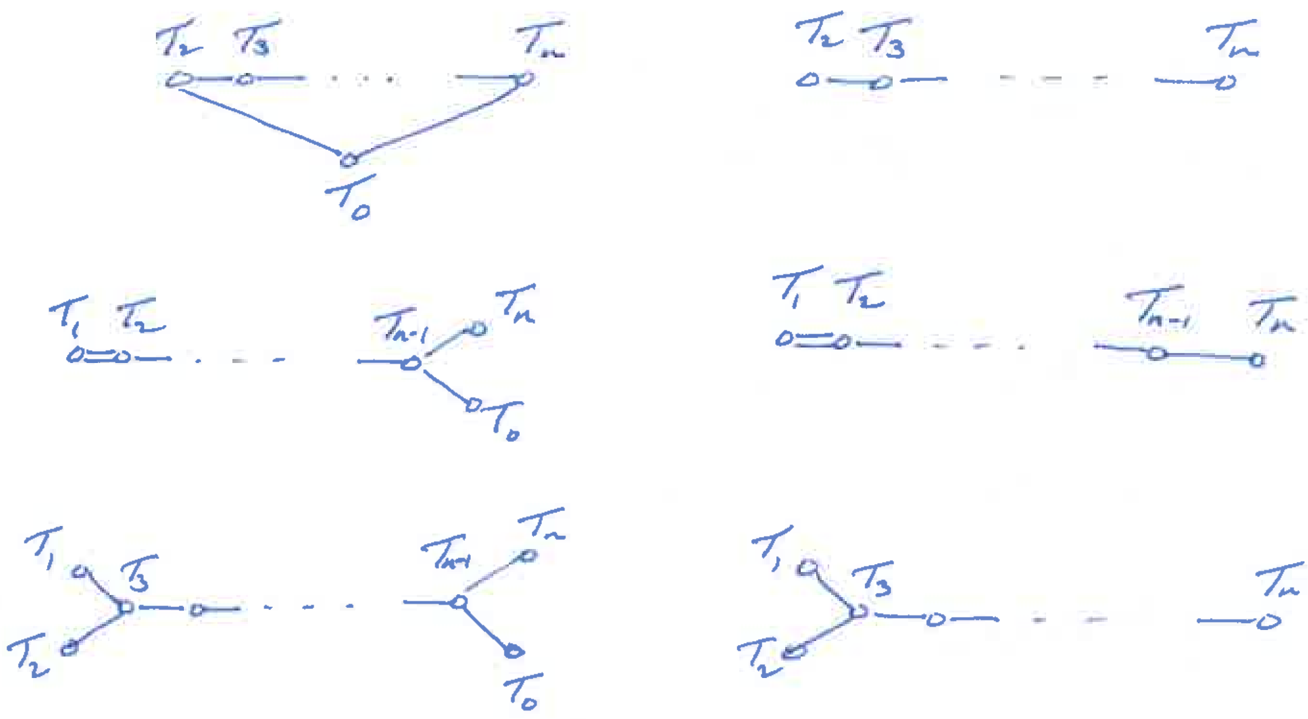
A. Ram and D. E. Taylor
"Explicit irreducible representations of the Iwahori-Hecke algebra of type F_4", preprint.

$$\left(\quad F_4 \text{ rep.} \quad \right)$$

Heckman and Opdam,

"Yang's System of Particles and graded Hecke algebras"

to appear in Ann. Math.



AFFINE HECKE ALGEBRAS

Generators: $T_1, T_2, \ldots, T_n, \quad T_0$

Relations: $\underbrace{T_i T_j T_i \cdots}_{m_{ij}} = \underbrace{T_j T_i T_j \cdots}_{m_{ij}}$

$$T_i^2 = (q - q^{-1}) T_i + 1$$

Bernstein-Zelevinsky-Lusztig

Generators: $T_1, T_2, \ldots, T_n, \quad y^\lambda, \quad \lambda \in P^\vee$

Relations:

$$\langle T_1, T_2, \ldots, T_n \rangle = H$$

$$\langle y^\lambda, \lambda \in P^\vee \rangle = \mathbb{C}[P^\vee]$$

$$y^\lambda y^\mu = y^\mu y^\lambda = y^{\mu+\lambda}$$

$$T_i y^\lambda = y^{s_i \lambda} T_i + (q - q^{-1}) \left(\frac{y^\lambda - y^{s_i \lambda}}{1 - y^{-\alpha_i^\vee}} \right)$$

Homomorphisms

$T_i \longmapsto T_i \qquad 1 \le i \le n$

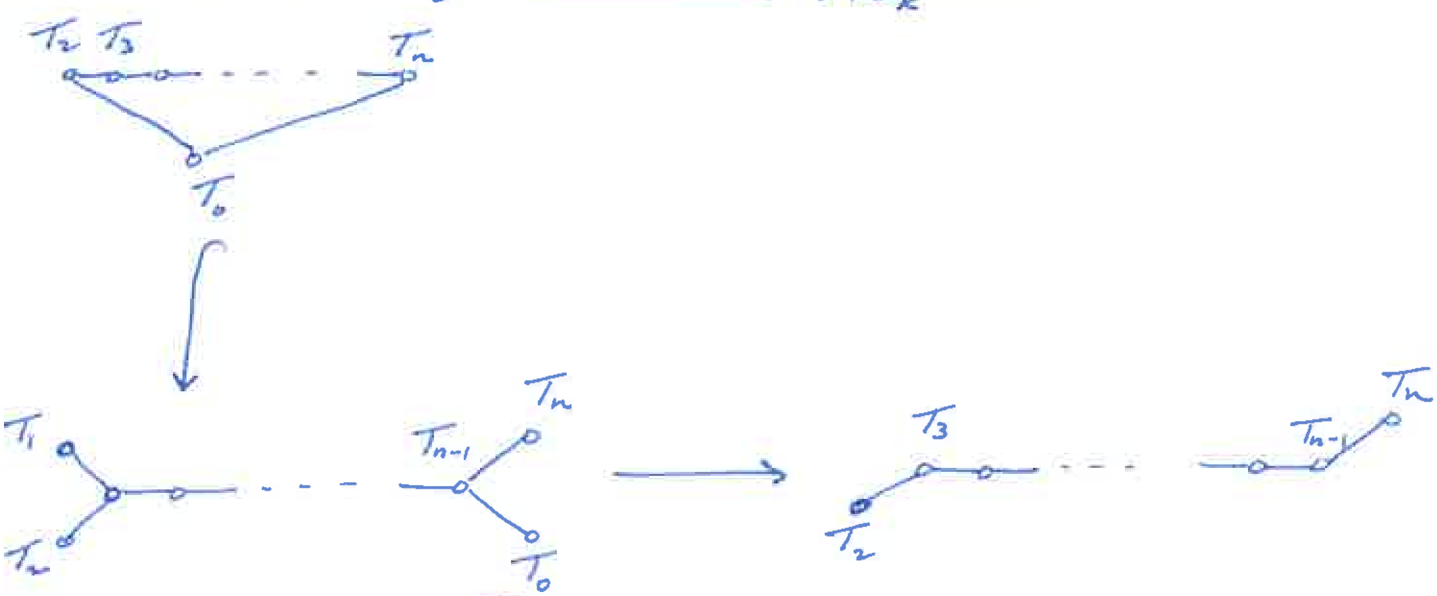
$T_0 \longmapsto T_n$

$y^{\varepsilon_k} \longmapsto MB_k$

$T_i \longmapsto T_i \qquad 1 \le i \le n$

$T_0 \longmapsto T_n$

$y^{\varepsilon_k} \longmapsto MD_k$



$y^{\varepsilon_k} \longmapsto MA_k$

Corollary Irreducible representations of

$$HA_{n-1}, \quad HB_n, \quad HD_n$$

are irreducible representations of

$$\hat{H}A_{n-1}, \quad \hat{H}B_n, \quad \hat{H}D_n .$$

An $\hat{H}$-module M is if

(2) M has a unique basis of simultaneous eigenvectors of Y^λ, $\lambda \in P^\vee$.

(3) Each weight ι is of the form

$$\iota(Y^\lambda) = q^{(\lambda,\mu)} \quad \text{for some } \mu \in \mathfrak{h}^*.$$

(2) means

$$Y^\lambda v_\iota = \iota(Y^\lambda) v_\iota \quad \text{where} \quad \begin{aligned} \iota: P^\vee &\longrightarrow \mathbb{C}^* \\ Y^\lambda &\longmapsto \iota(Y^\lambda) \end{aligned}$$

(3) means

$$\iota(Y^{\varepsilon_i}) = q^{\mu_i} \quad \text{for some } \mu_i \in \mathbb{R}.$$

Theorem **Type A** (Assume $\mu_i \in \mathbb{Z}$).

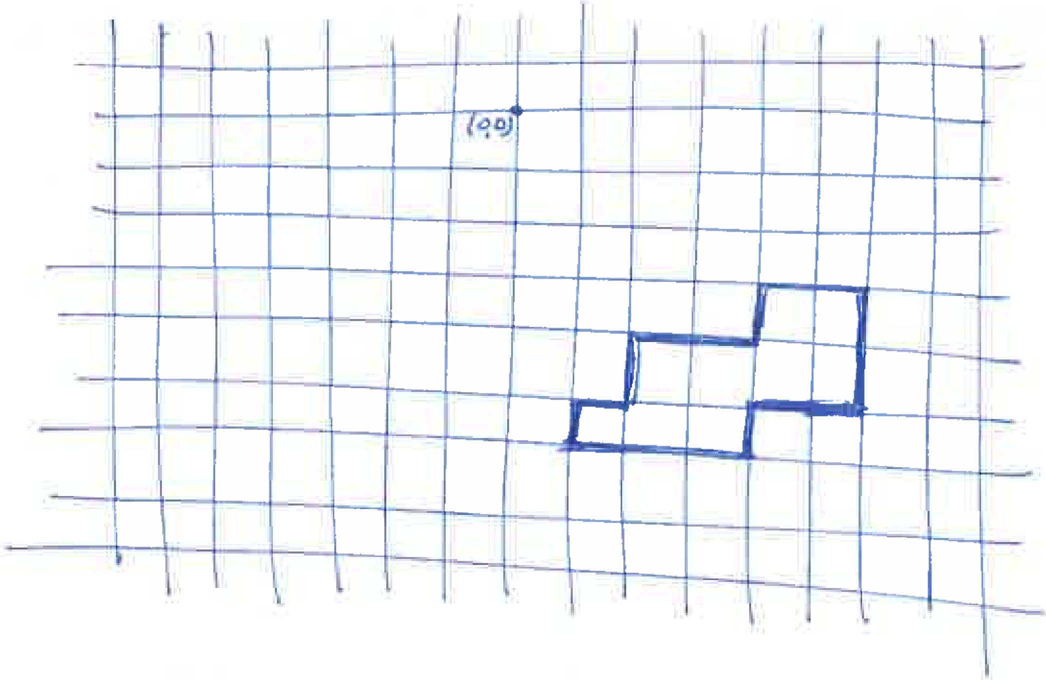
1. The irreducible representations are indexed by placed skew shapes
$$M^{(\lambda/\mu, P)}$$

2. **Basis:**
$$M^{(\lambda/\mu, P)} = \mathrm{span}\{ v_L \mid L \text{ standard of shape } \lambda/\mu \}.$$

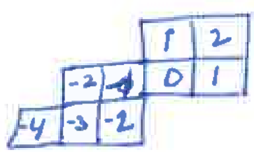
3.
$$y^{\varepsilon_i} v_L = q^{2c(L(i))} v_L .$$

4.
$$T_i v_L = (T_i)_{LL} v_L + (q^{-1} + (T_i)_{LL}) v_{s_i L}$$

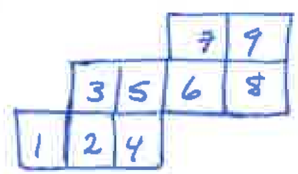
where
$$(T_i)_{LL} = \frac{q - q^{-1}}{1 - q^{2c(L(i-1)) - 2c(L(i))}}$$



Contents of Boxes



Ordering of boxes



$\pi = (-4, -3, -2, -2, -1, 0, 1, 1, 2)$

$Z(\pi) = \{ (i,j) \mid i > j,$ i,j are in the same diagonal $\}$.

$P(\pi) = \{ (i,j) \mid i > j,$ i,j are in adjacent diagonals $\}$

$J = \{ (i,j) \in P(\pi) \mid j$ is northeast of i in $\lambda/\mu \}$

Theorem Affine Hecke algebra $\hat{H}$ corresponding to a finite Weyl group W.

① Any irreducible representation has associated a unique (π, J) such that

$$M^{(\pi,J)} = \mathrm{span}\{ v_w \,/\, w \in W^\pi,\ R(w^{-1}) \cap P(\pi) = J \}$$

π is a dominant weight

$J \subseteq P(\pi)$

$P(\pi) = \{ \alpha > 0 \,/\, (\pi, \alpha^\vee) = 1 \}$

$R(w) = R^+ \cap wR^- = \{ \alpha > 0 \,/\, w^{-1}\alpha < 0 \}$.

W^π = minimal length representatives for W/W_π

W_π = stabilizer of π.

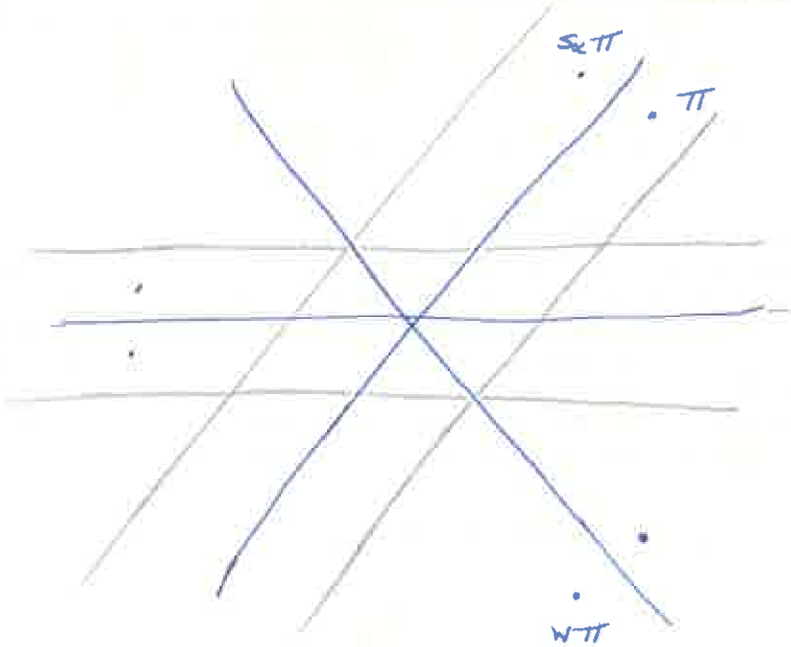
② $y^\mu v_w = q^{2(w\pi, \mu)} v_w$ for all $\mu \in P^\vee$

③ $T_i v_w = (T_i)_{ww} v_w + (q^{-1} + (T_i)_{ww}) v_{s_i w}$

where

$$(T_i)_{ww} = \frac{q - q^{-1}}{1 - q^{2(w\pi, -\alpha_i^\vee)}}$$

SHI ARRANGEMENT



Graph: $\Gamma(\pi)$

Vertices: $w \in W^{\pi}$

Edges: $w - s_i w$ if $(w\pi, \alpha_i^\vee) \neq \pm 1$.

Proposition: The connected components of $\Gamma(\pi)$ are

$$W^J = \{ w \in W^{\pi} / R(w^{-1}) \cap P(\pi) = J \}, \qquad J \subseteq P(\pi)$$

Ribbons and Descents

Assume π is regular i.e. in type A $\pi = (\pi_1 < \pi_2 < \cdots < \pi_n)$.

Then

$$P(\pi) \subseteq \{\text{simple roots}\} = \{(21), (32), (43), \ldots, (n, n-1)\}$$

$R(w^{-1}) \cap P(\pi) \subseteq D(w)$ the descent set of w.

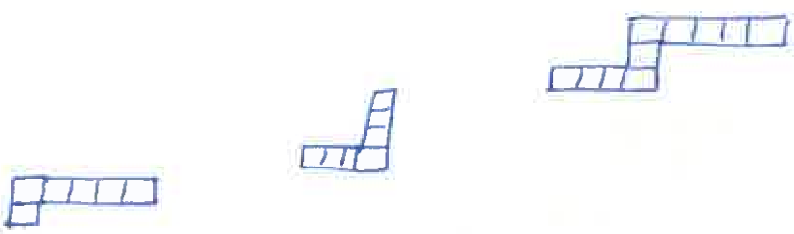
$W^{\pi} = W$ since $W_{\pi} = \{1\}$.

So

$$\mathcal{W}^J = \{ w \in W \,/\, D(w) \cap P(\pi) = J \}.$$

$$\Big\updownarrow \; 1\text{-}1$$

standard tableaux of ribbon shape



λ/μ has a break at i if $(i+1, i) \notin P(\pi)$

λ/μ has $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ at i if $(i+1, i) \in P(\pi)$, $(i+1, i) \in J$

λ/μ has $\square\square$ at i if $(i+1, i) \in P(\pi)$, $(i+1, i) \notin J$.

CONCLUSION

(1) Generalized "skew shape" to all types.

(2) Generalized "standard tableaux" to all types

(3) Generalized "ribbons" to all types.

(4) Described representations of affine Hecke algebras explicitly.

(5) Let $M^{(\lambda/\mu, \mathcal{B})}$ be a representation

Type A:

$$M^{(\lambda/\mu, \mathcal{B})} \Big\downarrow^{\hat{H}A_{n-1}}_{HA_{n-1}} = \sum_{\nu \vdash n} c^{\lambda}_{\mu\nu} S^{\nu}$$

$c^{\lambda}_{\mu\nu}$ is the LR coefficient.

General:

$$M^{(\pi, J)} \Big\downarrow^{\hat{H}}_{H} = \sum_{\nu} d^{(\pi, J)}_{\nu} H^{\nu}.$$

$d^{(\pi, J)}_{\nu}$ are a generalization of $c^{\lambda}_{\mu\nu}$ to all types.