Chapter 1. HOW TO DO PROOFS

There *is* a certain "formula" or method to doing proofs. Some of the guidelines are given below. The most important factor in learning to do proofs is practice, just as when one is learning a new language.

1) There are very few words needed in the structure of a proof. Organized in rows by synonyms they are:

```
To show
Assume, Let, Suppose, Define, If
Since, Because, By
Then, Thus, So
There exists, There is
Recall, We know, But.
```

2) The overall structure of a proof is a block structure like an outline. For example:

```
To show:  If __A__ then __B__ and __C__.
   Assume:  __A__.
   To show:  a) __B__.
             b) __C__.
      a)  To show:  __B__.
                _____ .
                _____ .
          Thus __B__.
      b)  To show:  __C__.
                _____ .
                _____ .
          Thus __C__.
   So __C__.
So, if __A__,then __B__ and __C__.
```

3) Any proof or section of proof begins with one of the following

   a) `To show:  If __A__ then __B__ .`
   b) `To show:  There exists __C__ such that __D__.`
   c) `To show:  __E__.`

   Immediately following this, the next step is

   Case a) Assume the ifs and 'to show' the thens. The next lines usually are

   ```
   Assume __A__.
   To show:  __B__.
   ```

   Case b) To show an object exists you must find it.
   The next lines usually are

   ```
   Define __xxx__.
   To show:  __xxx__ satisfies __D__.
   ```

   Case c) Rewrite the statement in __E__ by using a definition
   The next line is usually

   ```
   To show __E'__.
   ```

A useful guideline is, "Don't think too much." Following the "method" usually produces a proof without thinking. Most of doing proofs is simply rewriting what has come just before in a different form by plugging in a definition.

There are some kinds of proofs which have a special structure.

Proofs by contradiction.

(*) `Assume` the opposite of what you want to show.

_____ .
_____ .
_____ .

End up showing the opposite of some assumption (not necessarily the (*) assumption).
`Contradiction.`
`Thus` Assumption (*) is wrong and what you want to show is true.

Counterexamples.

To show that a statement, "If ____ then ____ ," is false you <u>must</u> give an example.

`To show: There exists a` _____ `such that`

a) it satisfies the `ifs` of the statement that you are showing is false.
b) it satisfies the opposite of some assertion in the `thens` of the statement that you are showing is false.

Proofs of uniqueness.

To show that an object is unique you must show that if there are two of them then they are really the same.

`To show:` A *THING* `is unique.`
`Assume` X_1 `and` X_2 `are both` *THING*`s.`
`To show:` $X_1 = X_2$.

Proofs by induction.

A statement to be proved by induction must have the form

`If` __A__ `then` __B__ `for all positive integers` n

The proof by induction should have the form

`Proof by induction.`
`Base case:`
`To show: If` __A__ `then` __B__ `for` __$n = 1$__
_____ .
_____ .
_____ .
`Thus if` __A__ `then` __B__ `for` __$n = 1$__
`Induction step:`
`Assume If` __A__ `then` __B__ `for` __$n < N$__ .
`To show: If` __A__ `then` __B__ .
This last to show line contains exactly the same statement except with n replaced by N and "for all positive integers n" removed.