Chapter 2. SPECIAL GROUPS OF LOW ORDER

In this chapter we shall give tables which give explicit information about several interesting examples of groups which have order less than 100. For the most part we shall not prove the results given in these tables. We strongly suggest that, in each individual case, the reader do the appropriate computations to check the information in these tables, for it is exactly in the computations in examples such as these that the subject of group theory "comes alive".

Let us begin with a list of the different groups of order ≤ 15. The reader should think about extending this table to include all groups of order, say, ≤ 100. The following beautiful book may be very helpful for such a project:

H.S.M. Coxeter and W.O.J. Moser, *Generators and Relations for Discrete Groups*, Series Ergebnisse der Mathematik und ihrer Grenzgebiete **14**, Springer-Verlag, Berlin 1984.

Note also that the finite abelian groups are completely determined by the Fundamental Theorem of Abelian groups, Theorem ().

In the following table: Q denotes the quaternion group.
$\mathbb{Z}_k$ denotes the cyclic group of order k.
D_k denotes the dihedral group of order $2k$.
S_k denotes the symmetric group on k letters.
A_k denotes the alternating group on k letters.

Group	Order	Abelian
(1)	1	Yes
$\mathbb{Z}_2$	2	Yes
$\mathbb{Z}_3$	3	Yes
$\mathbb{Z}_4$	4	Yes
$\mathbb{Z}_2 \times \mathbb{Z}_2$	4	Yes
$\mathbb{Z}_5$	5	Yes
$\mathbb{Z}_6 \simeq \mathbb{Z}_2 \times \mathbb{Z}_3$	6	Yes
$S_3 \simeq D_3$	6	No
$\mathbb{Z}_7$	7	Yes
$\mathbb{Z}_8$	8	Yes
$\mathbb{Z}_4 \times \mathbb{Z}_2$	8	Yes
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	8	Yes
D_4	8	No
Q	8	No
$\mathbb{Z}_9$	9	Yes
$\mathbb{Z}_3 \times \mathbb{Z}_3$	9	Yes
$\mathbb{Z}_{10} \simeq \mathbb{Z}_5 \times \mathbb{Z}_2$	10	Yes
D_5	10	No
$\mathbb{Z}_{11}$	11	Yes
$\mathbb{Z}_{12} \simeq \mathbb{Z}_4 \times \mathbb{Z}_3$	12	Yes
$\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	12	Yes
$D_6 \simeq \mathbb{Z}_2 \times D_3$	12	No
A_4	12	No
$\langle S, T \mid S^3 = T^2 = (ST)^2 \rangle$	12	No
$\mathbb{Z}_{13}$	13	Yes
$\mathbb{Z}_{14} \simeq \mathbb{Z}_7 \times \mathbb{Z}_2$	14	Yes
D_7	14	No
$\mathbb{Z}_{15} \simeq \mathbb{Z}_5 \times \mathbb{Z}_3$	15	Yes