MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 13: SELECTED ANSWERS

Problem A. Motion.

(2) (i) $s = 95$ m, $v = 53$ m/s, $a = 24$ m/s^2 (ii) 42 m

(3) (i) 1.5 sec (ii) $v = 2.5$ cm/s, $x = 4.5$ cm

(4) (i) $s = 18$ m and $a = 0$ (ii) $s = 26$ m, $v = 10$ m/s

(5) (i) $t = 0$, $t = 1$, $t = 3$,
(ii) velocities are 3 m/s, -2 m/s, 6 m/s, and
accelerations are -8 m/s^2, -2 m/s^2, 10 m/s^2

(10) 38 cm

(11) (i) $v = 0$, $a = -6$ m/s^2, (ii) $t = 1$ sec, $t = 2$ sec (iii) 6 m

(12) $v = 9$ cm/s, $a = 40$ cm/s^2

(13) $a = 1$, $b = 1/2$, $c = 2 - \pi/4$

(14) 122.5m (15) $v = 90.2$ m/s, $t = 10.2$ sec, $s = 510.2$ m

(16) $t = 4$ sec and $t = 6$ sec, $v = -29.4$ m/s, and it hits the ground at $t = 10$ sec

(17) 29.4 (18) 49 m (19) 122.5 m

Problem B. Applications of the exponential function.

(1) $y(t) = be^{k(t-a)}$

(6) (a) \$649.80 (b) \$658.40 (c) \$660.49 (d) \$661.53 (e) \$661.56 (f) \$661.56

(7) \$630.08 per month (8) \$25,167.03 (9) \$119.34

(10) \$1324.13 per month (11) \$1,984,172 (12) \$1204.01

(13) (a) approx. 137° F (b) after approx. 116 min

(14) approx. 2489 years (15) (a) approx. 3.82 days (b) approx. 12.68 days

(16) (a) $200 \cdot 2^{-t/140}$ mg (b) approx. 121.9 mg (c) approx. 605 days

(17) $N_0 e^k t$ (18) $Q_0 e^{kt}$ (19) $S + (T_0 - S)e^{kt}$

(20) In late 2025 (21) approx. 17.67 years (22) 95.8%

(23) 3.5 mg (24) 5×10^9 years (25) 1890 years

(26) 4800 years (27) 29.0 years (28) around 3060 BC

Problem C. Logarithmic differentiation.

(1) $\dfrac{dy}{dx} = \dfrac{-(x^2 - 4x - 42)(x+2)^{3/2}}{3(x+3)^{10/3}(x+6)^{3/2}}$.

(2) $\dfrac{dy}{dx} = y\left(\dfrac{2}{x+1} + \dfrac{3}{x-2} + \dfrac{1}{x+4} + \dfrac{1}{x \ln x}\right)$.

(3) $\dfrac{dy}{dx} = (y/2)\left(\dfrac{1}{x-a} + \dfrac{1}{x-b} - \dfrac{1}{x-p} - \dfrac{1}{x-q}\right)$.

(4) $\dfrac{dy}{dx} = (\sin x)^{\ln x}\left(\dfrac{1}{x}\ln\sin x + \cot x \ln x\right)$.

(5) $\dfrac{dy}{dx} = (\sin x)^{\cos x}(\cot x \cos x - \sin x \ln \sin x)$.

(6) $\dfrac{dy}{dx} = (\sin x)^{\tan x}(1 + \sec^2 x \ln \sin x) + (\tan x)^{\sin x}(\sec x + \cos x \ln \tan x)$.

(7) $\dfrac{dy}{dx} = \dfrac{-yx^{y-1} + y^x \ln y}{x^y \ln x + xy^{x-1}}$.

(8) $\dfrac{dy}{dx} = \dfrac{xy + y^2 - x}{x - x(x+y)\ln x}$.

(9) $\dfrac{dy}{dx} = \dfrac{\ln \sin y + y \tan x}{\ln \cos x - x \cot y}$.

(10) $\dfrac{dy}{dx} = a^x \ln a + e^{\tan x} \sec^2 x + (\cot x)^{\cos x}(\sin x \ln \tan x - \ln \csc x)$.

(11) $\dfrac{dy}{dx} = (\tan x)^{\cot x} \csc^2 x(1 - \ln \tan x)$.

(12) $\dfrac{dy}{dx} = x^x \ln(xe) + \dfrac{1}{x^2} x^{1/x} \ln(e/x)$.

(13) $\frac{dy}{dx} = (\sec x)^{\csc x}(\sec x - \csc x \cot x \ln \sec x) + (\csc x)^{\sec x}(\sec x \tan x \ln \csc x - \csc x).$

(14) $\frac{dy}{dx} = \frac{1}{x \ln ey}.$

(15) $\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \ln \cos x}.$

(16) $\frac{dy}{dx} = \frac{-y^2}{x(1 - y \ln x)}.$

(17) $\frac{dy}{dx} = \ln y + \frac{1}{x \ln x}.$

(18) $\frac{dy}{dx} = \frac{x^{1/x}(1 - \ln x)}{x^2}.$

(19) $\frac{dy}{dx} = \frac{1 + 2\ln x + x^{-2}}{(x^x + x^{-x})^{1/2}(x^x - x^{-x})^{3/2}}.$

Problem D. L'Hôpital's rule.

(6) 5	(7) a/b	(8) 1	(9) 0
(10) 1	(11) 0	(12) $-\infty$	(13) $\ln 3$
(14) 1/6	(15) $-1/6$	(16) 1/5	
(17) α	(18) 0	(19) 0	
(20) 0	(21) 0	(22) 1	(23) ∞
(24) 0	(25) 0	(26) 0	(27) 1
(28) e^{-2}	(29) e^3	(30) 1	(31) 1
(32) e^{-1}	(33) 1		