MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 14: SELECTED ANSWERS

Problem A. Derivatives with all functions mixed together.

(1) $\dfrac{dy}{dx} = \dfrac{2(\sec x + \tan x \sin x)}{(\tan x + \cos x)^3}$.

(2) $\dfrac{dy}{dx} = \dfrac{x \cos x \sin x}{2\sqrt{x \sin x}}$.

(3) $\dfrac{dy}{dx} = \dfrac{\cos 3x(1 + 2\cos 2x) + 3 \sin 3x(x + \sin 2x)}{\cos^2 3x}$.

(4) $\dfrac{dy}{dx} = 5e^{5x} \ln \sec x + e^{5x} \tan x$.

(5) $\dfrac{dy}{dx} = \dfrac{5x^4 \sin^{-1} 2x - 2x^5(1 - 4x^2)^{-1/2}}{(\sin^{-1} 2x)^2}$.

(6) $\dfrac{dy}{dx} = 2x \cos x^2 - \dfrac{(1 + x^2)\sec^2 x - 2x \tan x}{(1 + x^3)^2}$.

(7) $\dfrac{dy}{dx} = 3(\tan \sqrt{x} + x^2 - \sin x)^2 \left(\dfrac{\sec^2 \sqrt{x}}{2\sqrt{x}} + 2x - \cos x \right)$.

(8) $\dfrac{dy}{dx} = \dfrac{\sin^2 x(2 \cos 3x(3 \cos^2 x - \sec^2 x) + 3 \sin 2x \sin 3x)}{(\cos 3x)^2}$.

(9) $\dfrac{dy}{dx} = e^x(\sec^2 x + \tan x) + \dfrac{\sin x - x \cos x \ln x}{x \sin^2 x}$.

(10) $\dfrac{dy}{dx} = 2a^x(1/x + \ln x \ln a)$.

(11) $\dfrac{dy}{dx} = \dfrac{3x^2 - x^{3/2} - 1}{6x\sqrt{x}}$.

(12) $\dfrac{dy}{dx} = \dfrac{x^2 - 4x + 1}{(x^2 - x + 1)^2}$.

(13) $\dfrac{dy}{dx} = \dfrac{\ln \sin x + x \cot x \ln x}{2x\sqrt{1 + \ln x \ln \sin x}}.$

(14) $\dfrac{dy}{dx} = \dfrac{7}{2}x^{-1/2} - \dfrac{35}{2}x^{-9/2} + \dfrac{4x^3}{\sqrt{1-x^3}} + \dfrac{\csc^2 x}{\cos x}.$

(15) $\dfrac{dy}{dx} = 2 \sin x \cos^4 x - 3 \sin^3 x \cos^2 x.$

(16) $\dfrac{dy}{dx} = m \cos mx \cos nx - n \sin mx \sin nx.$

(17) $\dfrac{dy}{dx} = \sin^{m-1} x \cos^{n-1} x (m \cos^2 x - n \sin^2 x).$

(18) $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}.$

(19) $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{1-x^2}}.$

(20) $\dfrac{dy}{dx} = \dfrac{1}{2x^2}\left((\sqrt{a+x} - \sqrt{a-x})^2 - \dfrac{2x^2}{\sqrt{a^2-x^2}}\right).$

(21) $\dfrac{dy}{dx} = 96x^3 + 150x^2 + 70x + 10.$

(22) $\dfrac{dy}{dx} = -2\left(\tan\sqrt{1-x^2}\right)\left(\sec^2\sqrt{1-x^2}\right)\dfrac{x}{\sqrt{1-x^2}}.$

(23) $\dfrac{dy}{dx} = \dfrac{x - \sin 2x}{x^3 \cos^2 x}.$

(24) $\dfrac{dy}{dx} = \dfrac{e^{2x}(2x \ln x - 1)}{x}(\ln x)^2.$

(25) $\dfrac{dy}{dx} = e^{x^2}\left(1 + x^2\right)^{-3/2}\left(x(1 + 2x^2)\tan^{-1} x + 1\right).$

(26) $\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}+2}}{2\sqrt{x}} - \dfrac{e^{\sqrt{x+2}}}{2\sqrt{x+2}}.$

(27) $\dfrac{dy}{dx} = 2(\ln 7)(x+1)7^{x^2+2x}.$

(28) $\dfrac{dy}{dx} = -3\cot^2(e^{3x}x^x)\csc^2(e^{3x}x^x)e^{3x}x^x(4 + \ln x).$

(29) $\dfrac{dy}{dx} = \dfrac{1}{2(1+x^2)}$.

(30) $\dfrac{dy}{dx} = \dfrac{1}{(1+x^2)\tan^{-1} x}$.

(31) $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

(32) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$.

(33) $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

(34) $\dfrac{dy}{dx} = 0$.

(35) $\dfrac{dy}{dx} = \dfrac{-1}{1+x^2}$.

(36) $\dfrac{dy}{dx} = \sin^{-1} x$.

(37) $\dfrac{dy}{dx} = \cos^{-1} 2x$.

(38) $\dfrac{dy}{dx} = \dfrac{1}{5+3\cos x}$.

(39) $\dfrac{dy}{dx} = 1/2$.

(40) $\dfrac{dy}{dx} = \dfrac{\cos^{-1} x - x\sqrt{1-x^2}}{(1-x^2)^{3/2}}$.

(41) $\dfrac{dy}{dx} = \dfrac{2(a^2-x^2)-x}{4\sqrt{a^2-x^2}} + \dfrac{a^2}{2\sqrt{a^2-x^2}}$.

(42) $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}$.

(43) $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

(44) $\dfrac{dy}{dx} = 2x^2\cos 2x + 2x\sin 2x - \dfrac{(x+1)\sin x + \cos x}{(x+1)^2}$.

(45) $\dfrac{dy}{dx} = \dfrac{2x - x^4}{(x^2+1)^2} + 2x^4 \cos 2x + 4x^3 \sin 2x.$

(46) $\dfrac{dy}{dx} = (2 \ln x) x^{\ln x - 1}.$

(47) $\dfrac{dy}{dx} = (\tan x)^{\cot x} (\csc^2 x)(1 - \ln \tan x).$

(48) $\dfrac{dy}{dx} = x^{\cos^{-1} x} \left(\dfrac{\cos^{-1} x}{x} - \dfrac{\ln x}{\sqrt{1-x^2}} \right) + (\tan x)^{\cot x} (\csc^2 x)(1 - \ln \tan x)$
$+ (\cot x)^{\tan x} (\sec^2 x)(\ln \cot x - 1).$

(49) $\dfrac{dy}{dx} = (\cos x)(e^x)(\ln x)(x^x)(x^{\cos^{-1} x}) + (\sin x)(e^x)(\ln x)(x^x)(x^{\cos^{-1} x})$
$+ (\sin x)(e^x)(1/x)(x^x)(x^{\cos^{-1} x}) + (\sin x)(e^x)(\ln x)(1 + \ln x)(x^x)(x^{\cos^{-1} x})$
$+ (\sin x)(e^x)(\ln x)(x^x) + x^{\cos^{-1} x} \left(\dfrac{\cos^{-1} x}{x} - \dfrac{\ln x}{\sqrt{1-x^2}} \right).$

(50) $\dfrac{dy}{dx} = \dfrac{\ln y - (y/x)}{\ln x - (x/y)}.$

(51) $\dfrac{dy}{dx} = -\left(\dfrac{y}{x}\right)^3.$

(52) $\dfrac{dy}{dx} = -\dfrac{y}{x}.$

(53) $\dfrac{dy}{dx} = \dfrac{\cos(x+y) - y}{x - \cos(x+y)}.$

(54) $\dfrac{dy}{dx} = -\left(\dfrac{b}{a}\right)^m \left(\dfrac{x}{y}\right)^{m-1}.$

(55) $\dfrac{dy}{dx} = y/x.$

(56) $\dfrac{dy}{dx} = \dfrac{x - y}{x(1 + \ln x)}.$

Problem B. Integrals with mixed functions.

(1) $(1/2)(\sin^{-1} x)^2 + c$

(2) $(-1/2)\cos(2 \tan^{-1} x) + c$

(3) $\sin(\ln x) + c$

(4) $-\cot(\ln x) + c$

(5) $e^{\tan x} + c$

(6) $-e^{\cos^2 x} + c$

(7) $(1/2)(\ln(\sin x))^2 + c$

(8) $\ln|\ln\sin x| + c$

(9) $(1/2)(\ln(\sec x + \tan x))^2 + c$

(10) $(1/4)(\tan^{-1} x^2)^2 + c$

(11) $(1/4)(\sin^{-1} x^2)^2 + c$

(12) $\ln|\sin^{-1} x| + c$

(13) $\ln|x + \ln(\sec x)| + c$

(14) $\ln|\ln(\tan x)| + c$

(15) $\tan(1 + \ln x) + c$

(16) $(1/2)\cot(2e^{-x} + 5) + c$

(17) $(1/3)\sin(e^{x^3}) + c$

(18) $\dfrac{e^{m\tan^{-1} x}}{m} + c$

(19) $\tan(xe^x) + c$

(20) $2\sin(e^{\sqrt{x}}) + c$

Problem C. Areas of regions.

(1) πab sq. units

(2) $15/2$ sq. units

(3) 6π sq. units

(4) $2\sqrt{2}/3 + 9\pi/2 - 9\sin^{-1}(1/3)$ sq. units

(5) $\pi/2 + 1/3$ sq. units

(6) $\pi a^2/4 - 2a^2/3$ sq. units

(7) $\sqrt{2}/6 + 9\pi/8 - (9/4)\sin^{-1}(1/3)$ sq. units

(8) $\pi/4 - 1/2$ sq. units

(9) $23/6$ sq. units

(10) $1/3$ sq. units

Problem D. Different types of volume problems.

(1) $f(x) = \sqrt{\dfrac{2x - a}{\pi}}$

(2) $f(x) = \pm\sqrt{\dfrac{2x + 1}{\pi}}$

(3) $32\pi/3$ cubic units

(4a) $2\pi a^3$ cubic units

(4b) $16\pi a^3/15$ cubic units

(4c) $8\pi a^3/5$ cubic units

(5) $(8\pi/3)(2-\sqrt{2})$ cubic units

(6) $(\pi/15)(88\sqrt{2}+107)$ cubic units

(7) $112\pi a^3/15$ cubic units

(8) hs^2 cubic units

(9) hs^2 cubic units

(10) $8r^3/3$ cubic units

(11) $\pi^2/4$ cubic units

(12) $28\pi/3$ cubic units