HOMEWORK 2: SELECTED ANSWERS

Problem B. Computing some derivatives

(1) $\dfrac{dy}{dx} = 20x + 27.$

(2) $\dfrac{dy}{dx} = \dfrac{3}{2}x^{1/2} + \dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}x^{-3/2} - \dfrac{3}{2}x^{-5/2}.$

(3) $\dfrac{dy}{dx} = (2x - 5)(3x - 4)^2(30x - 61).$

(4) $\dfrac{dy}{dx} = 2ex - \dfrac{3\pi}{x^4} + \dfrac{7}{2}x^{5/2}.$

(5) $\dfrac{dy}{dx} = -\dfrac{2(x - 3)}{(x - 4)^3}.$

(6) $\dfrac{dy}{dx} = \dfrac{3x^2 + 10x + 12}{(4 - x^2)^2}.$

(7) $\dfrac{dy}{dx} = \dfrac{1 - x}{(1 - 2x)^{3/2}}.$

(8) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}(1 - \sqrt{x})^2}.$

(9) $\dfrac{dy}{dx} = -\dfrac{1}{(x - 1)^2} - \dfrac{1}{(x + 3)^2}.$

(10) $\dfrac{dy}{dx} = \dfrac{a}{\sqrt{a^2 - x^2}(a + \sqrt{a^2 - x^2})}.$

(11) $\dfrac{dy}{dx} = \dfrac{x^2 + 2x + 2}{(x + 1)^2}.$

(12) $\dfrac{dy}{dx} = \dfrac{-3}{2\sqrt{x}(x - 3)^{3/2}}.$

(13) $\dfrac{dy}{dx} = \dfrac{-2nx^{n-1}}{(x^n - 1)^2}.$

(14) $\dfrac{dy}{dx} = \dfrac{2x}{(1 - x^2)(1 - x^4)^{1/2}}.$

(15) $\dfrac{dy}{dx} = \dfrac{4x^2 + 1}{x^2(x^2 + 1)^{3/2}}.$

(16) $\dfrac{dy}{dx} = nu^{n-1}\dfrac{du}{dx}$.

(17) $\dfrac{dy}{dx} = \dfrac{-x}{\sqrt{1-x^2}}$.

Problem C. Correcting derivative identities

(1) $\dfrac{d}{dx}(x^{3/2}) = \dfrac{3}{2}x^{1/2}$.

(2) $\dfrac{d}{dx}(x^3 + 3) = 3x^2$.

(3) $\dfrac{d}{dx}(x+3)^{5/2} = \dfrac{5}{2}(x+3)^{3/2}$.

(4) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$.

(5) $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$.

(6) $\dfrac{d}{dx}(u \cdot v) = u\dfrac{dv}{dx} + \dfrac{du}{dx}v$.

Problem E. Derivatives at a point

(1) $\left.\dfrac{dy}{dx}\right|_{x=2} = 5$.

(2) $\left.\dfrac{dy}{dx}\right|_{x=2} = 5$.

(3) $\left.\dfrac{dy}{dx}\right|_{x=3} = 1476$.

(4) $\left.\dfrac{dy}{dx}\right|_{x=3} = 9$.

Problem F. Derivatives with respect to functions

(1) $\dfrac{dy}{dx} = \dfrac{2t^4 + 8}{5t^7}$.

(2) $\dfrac{dy}{dx} = \dfrac{1}{(1+x^2)^2}$.

(3) $\dfrac{dy}{dx} = \dfrac{(ad - bc)(c_1 x + d_1)^2}{(a_1 d_1 - b_1 c_1)(cx + d)^2}.$

(4) $\dfrac{dy}{dx} = \dfrac{3}{2}x.$

(5) $\dfrac{dy}{dx} = \dfrac{-1}{x^2(1 + \sqrt{1 - x^4})}.$

(6) $\dfrac{dy}{dx} = \dfrac{1 - x^2}{3x^2(1 + x^2)^2}.$

(7) $\dfrac{dy}{dx} = \dfrac{x + \sqrt{1 - x^2}}{-x}.$

(8) $\dfrac{dy}{dx} = \dfrac{35x^4 - 22x}{14x - 15}.$

Problem G. Derivatives of parametric equations

(1) $\dfrac{dy}{dx} = \dfrac{-1}{t^2}.$

(2) $\dfrac{dy}{dx} = 1/t.$

(3) $\dfrac{dy}{dx} = -1.$

(4) $\dfrac{dy}{dx} = \dfrac{b(t^2 - 1)}{2at}.$

(5) $\dfrac{dy}{dx} = \dfrac{t^3 + 2t - t^{-1}}{2}.$

(6) $\dfrac{dy}{dx} = \dfrac{b(1 + t^2)}{2at}.$

(7) $\dfrac{dy}{dx} = \dfrac{t(2 - t^3)}{1 - 2t^3}.$

(8) $\dfrac{dy}{dx} = -x/y.$

Problem H. Implicit differentiation

(1) $\dfrac{dy}{dx} = \dfrac{x(2a^2y^2 - x^2)}{y(y^2 - 2a^2x^2)}.$

(2) $\dfrac{dy}{dx} = \dfrac{-b^2x}{a^2y}.$

(3) $\dfrac{dy}{dx} = \dfrac{2axy^2 - x^4}{y^4 - 2ax^2y}.$

(6) $\dfrac{dy}{dx} = -\dfrac{ax + hy + g}{hx + by + f}.$