

MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 4: SELECTED ANSWERS

Problem A. Derivatives with exponentials, logs and trig functions.

- (1) $\frac{dy}{dx} = 0$.
- (2) $\frac{dy}{dx} = m \cot x + n \tan x$.
- (3) $\frac{dy}{dx} = e^{ax}(a \sin bx + b \cos bx)$.
- (4) $\frac{dy}{dx} = 2 \csc x$.
- (5) $\frac{dy}{dx} = \sec 2x$.
- (6) $\frac{dy}{dx} = e^{ax}(a \cos(bx + c) - b \sin(bx + c))$.
- (7) $\frac{dy}{dx} = \frac{x(1 + 2 \csc 2x) - 2(x + \ln \tan x)(1 + 2x)}{2(\sqrt{x + \ln \tan x})(x^2 e^{2x})}$.
- (8) $\frac{dy}{dx} = \frac{2(x \cos x + \sin x)}{1 - x^2 \sin^2 x}$.
- (9) $\frac{dy}{dx} = \csc x$.
- (10) $\frac{dy}{dx} = \sec x$.

Problem B. Derivatives at a point.

- (1) $\frac{dy}{dx} \Big|_{x=\pi/4}$ is undefined.
- (2) $\frac{dy}{dx} \Big|_{x=0} = 0$ and $\frac{dy}{dx} \Big|_{x=\pi/2} = -4 \left(\frac{2 + \pi}{4 + \pi^2} \right)^2$.
- (3) $\frac{dy}{dx} \Big|_{x=\pi/6} = \frac{8\sqrt{3}}{9} - 4$.
- (4) $\frac{dy}{dx} \Big|_{x=\pi/3} = -\frac{2\pi}{3} \sin(\sin(\pi^2/9)) \cos(\pi^2/9)$.

$$(5) \left. \frac{dy}{dx} \right|_{x=\pi^2/16} = \frac{27}{\pi}.$$

$$(6) \left. \frac{dy}{dx} \right|_{x=\pi/2} = 0.$$

$$(7) \left. \frac{dy}{dx} \right|_{x=0} = 0 \text{ and } \left. \frac{dy}{dx} \right|_{x=\pi/2} = -2.$$

$$(8) \left. \frac{dy}{dx} \right|_{x=0} = 0 \text{ and } \left. \frac{dy}{dx} \right|_{x=\sqrt{\pi/2}} = \frac{-2\sqrt{\pi}}{(2+\pi)^{3/2}}.$$

$$(9) \left. \frac{dy}{dx} \right|_{x=\pi/4} = 42 - \frac{105\sqrt{2}}{4}.$$

Problem D. Parametric equations.

$$(1) \frac{dy}{dx} = (-b/a) \cot \theta.$$

$$(2) \frac{dy}{dx} = \tan(\theta/2).$$

$$(3) \frac{dy}{dx} = \frac{3b}{2a} \tan \theta.$$

$$(4) \frac{dy}{dx} = -\cot \phi.$$

$$(5) \frac{-1}{4a} \csc^4(t/2) \quad (6) -1/a \quad (7) -3/2$$

Problem E. Implicit differentiation.

$$(1) \frac{dy}{dx} = \frac{(1+x^2)\sin x - (y^2+2x)\cos x - y\sec^2 x}{2y\sin x + \tan x}.$$

$$(2) \frac{dy}{dx} = \frac{y(2xy - y^2 \cos(xy) - 1)}{xy^2 \cos(xy) + y^2 - x}.$$

$$(3) \frac{dy}{dx} = \frac{2xy - 2(1+x^2)^2 \tan x \sec^2 x}{(1+x^2)^2(4y + \cos y) + (1+x^2)}.$$

$$(4) \frac{dy}{dx} = \frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)}.$$

$$(5) \frac{dy}{dx} = \frac{-y(ay^2 \cos(xy) - b \sin(x/y))}{x(ay^2 \cos(xy) + b \sin(x/y))}.$$

$$(6) \frac{dy}{dx} = \tan^{-1} e^x + \frac{xe^x}{1 + e^{2x}}.$$

Problem F. Derivatives with inverse trig functions.

$$(1) \frac{dy}{dx} = \frac{3x^2}{\sqrt{1-x^6}}.$$

$$(2) \frac{dy}{dx} = \frac{4x^3}{\sqrt{1-x^8}}.$$

$$(3) \frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+x)}.$$

$$(4) \frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}.$$

$$(5) \frac{dy}{dx} = \frac{5}{1+25x^2}.$$

$$(6) \frac{dy}{dx} = \frac{2}{x\sqrt{x^4-1}}.$$

$$(7) \frac{dy}{dx} = \frac{-2}{x\sqrt{x^4-1}}.$$

$$(8) \frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}}.$$

$$(9) \frac{dy}{dx} = \frac{4x}{\sqrt{1-4x^2}}.$$

$$(10) \frac{dy}{dx} = \frac{-\cot x}{\sqrt{\sin^2 x - 1}}.$$

$$(11) \frac{dy}{dx} = \frac{1}{2x\sqrt{x-1}}.$$

$$(12) \frac{dy}{dx} = \frac{\cos(\tan^{-1} x)}{1+x^2}.$$

$$(13) \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}} + \cos^{-1} x.$$

$$(14) \frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x.$$

$$(15) \frac{dy}{dx} = \frac{x}{1+x^2} + \tan^{-1} x.$$

$$(16) \frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}.$$

$$(17) \frac{dy}{dx} = 2x \tan^{-1} x + 1.$$

$$(18) \frac{dy}{dx} = \frac{-\tan x}{\sqrt{1-x^2}} + \cos^{-1} x \sec^2 x.$$

$$(19) \frac{dy}{dx} = \frac{2}{1-x^4}.$$

$$(20) \frac{dy}{dx} = -\sqrt{1-x^2} - 2x \cos^{-1} x.$$

$$(21) \frac{dy}{dx} = \frac{\tan x}{1+x^2} + \tan^{-1} x \sec^2 x.$$

$$(22) \frac{dy}{dx} = 0.$$

$$(23) \frac{dy}{dx} = \left(\frac{-a}{x^2 + a^2} \right) (-\tan^{-1}(a/x) + \cot^{-1}(x/a)).$$

$$(24) \frac{dy}{dx} = \frac{6(\tan^{-1} 2x)^2}{1+4x^2}.$$

$$(25) \frac{dy}{dx} = \frac{2x \sec^2 x}{\sqrt{1-\tan^2 x^2}}.$$

$$(26) \frac{dy}{dx} = \frac{2}{1+x^2}.$$

$$(27) \frac{dy}{dx} = \frac{-2}{1+x^2}.$$

$$(28) \frac{dy}{dx} = \frac{3}{(1+x^2)^2}.$$

$$(29) \quad \frac{dy}{dx} = \frac{2x}{1+x^4}.$$

$$(30) \quad \frac{dy}{dx} = \frac{2}{1+x^2}.$$

$$(31) \quad \frac{dy}{dx} = 1/2.$$

$$(32) \quad \frac{dy}{dx} = 3/2.$$

$$(33) \quad \frac{dy}{dx} = \frac{-(b^2 - a^2) \sin x}{b + a \cos x}.$$

$$(34) \quad \frac{dy}{dx} = \frac{\sqrt{3}}{2 + \cos x}.$$

$$(35) \quad \frac{dy}{dx} = 1/2.$$

$$(36) \quad \frac{dy}{dx} = 1.$$