MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 7: SELECTED ANSWERS

Problem A. Graphing rational functions.

(1) Defined for $x \neq 0$; continuous for $x \neq 0$; differentiable for all $x \neq 0$; increasing nowhere; decreasing for all $x \neq 0$; concave up for $x > 0$; concave down for $x < 0$; critical point at $x = 0$; no point of inflection; asymptote $y = 0$ as $x \to 0$; asymptote $x = 0$ as $x \to \pm\infty$.

(2) Defined for $x \neq 0$; continuous for $x \neq 0$; differentiable for all $x \neq 0$; increasing for $x > 0$; decreasing for all $x < 0$; concave up nowhere; concave down for $x \neq 0$; critical points at $x = 0$; no points of inflection; asymptote $y = 0$ as $x \to 0$;

(3) Defined for $x \neq 0$; continuous for $x \neq 0$; differentiable for all $x \neq 0$; increasing for $x < -1, x > 1$; decreasing for all $-1 < x < 0, 0 < x < 1$; concave up for $x > 0$; concave down for $x < 0$; critical points at $x = 0, \pm 1$; no points of inflection; asymptote $x = 0$ as $x \to 0$; asymptote $y = x$ as $x \to \pm\infty$;

(4) Defined for $x \neq 4$; continuous for $x \neq 4$; differentiable for all $x \neq 4$; increasing for $x < 2, x > 6$; decreasing for all $2 < x < 4, 4 < x < 6$; concave up for $x > 4$; concave down for $x < 4$; critical points at $x = 2, 4, 6$; no points of inflection; asymptote $x = 4$ as $x \to 4$; asymptote $y = x$ as $x \to \pm\infty$;

(5) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 0$; decreasing for all $x > 0$; concave up for $x < -1/\sqrt{3}, x > 1/\sqrt{3}$; concave down for $-1/\sqrt{3} < x < \sqrt{3}$; critical point at $x = 0$; points of inflection at $x = \pm 1/\sqrt{3}$; asymptote $y = 0$ as $x \to \pm\infty$;

Problem B. Graphing functions with square roots.

(1-5) Circles.

(6-8) Ellipses.

(9-10) Hyperbolas.

(11-16) Parabolas.

(18) This problem appeared before on this homework assignment (almost).

(19) Make this one into problem (B18).

Problem C. Graphing other functions.

(1) Defined for all x; continuous for all $x \neq 0, \pm 1, \pm 2, \ldots$; differentiable for all $x \neq 0, \pm 1, \pm 2, \ldots$; increasing nowhere; decreasing nowhere; concave up nowhere; concave down nowhere; all points are critical points; no points of inflection; no asymptotes;

(2) Defined for all x; continuous for all x; differentiable for $x \neq 0$; increasing for $x > 0$; decreasing for $x < 0$; concave up nowhere; concave down nowhere; critical point at $x = 0$; no points of inflection; asymptote $y = x$ as $x \to \infty$; asymptote $y = -x$ as $x \to -\infty$;

(3) Defined for all x; continuous for all x; differentiable for $x \neq 5$; increasing for $x > 5$; decreasing for $x < 5$; concave up nowhere; concave down nowhere; critical point at $x = 5$; no points of inflection; asymptote $y = x$ as $x \to \infty$; asymptote $y = -x$ as $x \to -\infty$;

(4) Defined for all x; continuous for all x; differentiable for $x \neq \pm 1$; increasing for $-1 < x < 0$, $x > 1$; decreasing for $x < -1$, $0 < x < 1$; concave up for $x < -1$, $x > 1$; concave down for $-1 < x < 1$; critical points at $x = \pm 1, 0$; no points of inflection; no asymptotes;

(5) Defined for all x; continuous for all $x \neq 0$; differentiable for $x \neq 0$; increasing nowhere; decreasing nowhere; concave up nowhere; concave down nowhere; critical point at $x = 0$; no points of inflection; asymptotes $y = 1$ as $x \to \infty$; asymptotes $y = -1$ as $x \to -\infty$;

(6) Defined for all x; continuous for all x; differentiable for $x \neq 1$; increasing for all x; decreasing nowhere; concave up for $x < 1$; concave down for $x > 1$; critical point at $x = 1$; point of inflection at $x = 1$; no asymptotes;

(7) Defined for all x; continuous for all x; differentiable for $x \neq 0$; increasing for $x > 0$; decreasing for $x < 0$; concave up nowhere; concave down everywhere; critical point at $x = 0$; no points of inflection; no asymptotes;

(8) Defined for $x \neq 1$; continuous for all $x \neq 1$; differentiable for $x \neq 1$; increasing for $x < 1$; decreasing for $x > 1$; concave up everywhere; concave down nowhere; critical point at $x = 1$; no points of inflection; no asymptotes;

(12) See page 153 in the text.

(13) Make this one into problem (C12).

(14) Defined for all x; continuous for all x; differentiable for x; increasing for $2k\pi + -\pi/2 < x < 2k\pi + \pi/2$, where k is an integer; decreasing for $2k\pi + \pi/2 < x < 2k\pi + 3\pi/2$, where k is an integer; concave up for $2k\pi - \pi < x < 2k\pi$, where k is an integer; concave down

for $2k\pi < x < 2k\pi + \pi$, where k is an integer; critical points at $x = k\pi + \pi/2$, where k is an integer; points of inflection at $x = k\pi$, where k is an integer; no asymptotes;

(15) Compare the graphs of $y = \sin 2x$ and $y = x$.

Problem D. Rolle's theorem and the mean value theorem.

(4) $c = 2 \pm \sqrt{3}/3$ (5) $c = 9/4$ (6) $c = 3\pi/2$

(7) $c = \pi/4$ (8) $c = 2 \pm \sqrt{3}/3$ (11) $f(1) \neq f(3)$

(12) $f'(1)$ does not exist (13) $f(x)$ is discontinuous at $x = 0$

(14) $(3, 6)$ (17) $c = 8/27$ (18) $c = e - 1$ (20) $c = (a + b)/2$

(21) $f'(0)$ does not exist (22) $f(x)$ is discontinuous at $x = 0$

(23) $(\sqrt{7/3}, (-2/3)(\sqrt{7/3}))$, $(-\sqrt{7/3}, (2/3)(\sqrt{7/3}))$

Problem E. Tangents and Normals.

(1) 11 (2) -12

(3) $x - y + 5 = 0, \quad x + y - 7 = 0$

(4) $x - 4y + 3 = 0, \quad 4x + y - 5 = 0$

(5) $14x - y - 10 = 0, \quad x + 14y - 254 = 0$

(6) $m^2x - my + a = 0, \quad m^2x + m^3y - 2am^2 - a = 0$

(7) $bx\cos\theta + ay\sin\theta = ab, \quad ax\sec\theta - by\csc\theta = (a^2 - b^2)$

(8) $bx\sec\theta - ay\tan\theta = ab, \quad x\sin\theta - y\cos\theta = (a/4)\sin 4\theta$

(9) $x\cos^3\theta + y\sin^3\theta = c, \quad x\sin^3\theta - y\cos^3\theta + 2c\cot 2\theta = 0$

(10) $x - ty + at^2 = 0, \quad tx + y = at^3 + 2at$

(11) $2x + 3my - 18m^2 - 27m^4 = 0$