

MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 8: SELECTED ANSWERS

Problem A. Tangents and normals.

- (1) $10x + y - 5 = 0, \quad x - 10y + 50 = 0$
- (2) $y = 1, \quad x = \pi/4$
- (3) $y - 1 - (1/\sqrt{2}) = (1 - \sqrt{2})(x - \pi/4 - 1/\sqrt{2})$
- (4) $x/a + y/b = \sqrt{2}$
- (5) $bx \cos t + ay \sin t = ab, \quad ax \sec t - by \csc t = (a^2 - b^2)$
- (6) $bx \sec t - ay \tan t = ab, \quad ax \cos t + by \cot t = (a^2 + b^2)$
- (7) $y = (x - at) \tan(t/2), \quad (y - 2a) \tan(t/2) + x - at = 0$
- (8) $8x + 3\sqrt{5}y - 36 = 0, \quad 9\sqrt{5}x - 24y + 14\sqrt{5}$
- (9) $3y - 48\sqrt{3}x + 16\pi\sqrt{3} - 21 = 0$
- (10) $24y - 2x + \pi - 96 = 0$
- (11) $x + (2 + \sqrt{2})yi - 4 - 3\sqrt{2} - \pi/4 = 0$
- (14) $(0, 0), (2a, -2a)$
- (16) $(0, 0), (1, 2), (-1, -2)$
- (17) $(2, 3), (-2, -3)$
- (18) $(1, 7)$
- (19) $x + 3y - 9 = 0$
- (20) $2x + y - 6 = 0$

Problem B. Optimization.

- (1) $(1/5, 4)$ (2) $(3, 9)$ (3) $(-4, 6)$
- (4) $(k\pi + \pi/4, 6)$ and $(k\pi + 3\pi/4, 4)$ where k is an integer
- (5) $(k\pi/2 + \pi/8, 4)$ and $(k\pi/2 + 3\pi/8, 2)$ where k is an integer

- (6) $(1, 68), (-6, -1647), (5, -316)$ (7) $(-2, 0), (0, -4)$
 (8) $(1, 0), (-1, 0), (-1/5, -3456/3125)$ (9) $(2, 2)$
 (10) $(-2, 139), (3, 89)$ (11) $(\pi/6, 3/4), (\pi/2, 1/2)$
 (14) 49 (15) $(1, 3)$
 (16) $(\pi/3, -\pi/3 + \sqrt{3}), (5\pi/3, -(5\pi/3 + \sqrt{3}))$
 (17) 6, 9 (18) $\frac{ap}{p+q}, \frac{aq}{p+q}$ (20) $(1/p)^{1/(p-1)}$
 (21) length $4\sqrt{6}$ cm, width $4\sqrt{6}$ cm, perimeter $16\sqrt{6}$ cm
 (25) $r = (500/\pi)^{1/3}$ cm, $h = \frac{1000}{\pi^{1/3}500^{1/3}}$ cm
 (26) $p^3/6\sqrt{3}$ cubic units (28) 4, 4 (29) 75×75 yards

Problem C. Related rates.

- (1) $\frac{dV}{dr} = 4\pi r^2$ (2) $\frac{dV}{dr} = 2\pi r h$
 (3) $\frac{dS}{dr} = \frac{\pi(h^2 + 2r^2)}{\sqrt{h^2 + r^2}}$ (4) $\frac{dP}{dt} = 0.8$ cm/s
 (5) $\frac{dr}{dt} \approx 0.32$ cm/s (6) $\frac{dV}{dt} = 6$ cm³/s
 (7) $\frac{dh}{dt} = 0.5$ cm/min (8) 6.78 m/s
 (9) (a) $25/3$ ft/s (b) $10/3$ ft/s (10) $80\pi/3$ km/min
 (11) $\sqrt{65}/8$ m/sec (12) $6/5\pi$ ft/min
 (13) $32/27\pi$ cm/s (14) $\approx 2.89 \times 10^5$ cm³/min
 (15) $16/49\pi$ cm/sec (16) $\sqrt{2}/5$ rad/sec
 (17) $5/6$ m/s (18) 0.032 rad/s
 (19) $\pi/3$ radians (20) (a) 360 ft/s (b) 0.096 rad/s