MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 3
DUE September 27, 2004

Problem A. Vocabulary and basic identities.

(1) What is $\dfrac{d}{dx}$?

(2) What is e^x?

(3) What is $\ln x$?

(4) What is $\sin x$?

(5) What is $\cos x$?

(6) Explain why $f(x) = e^x$ is the only function such that $\dfrac{df}{dx} = f$ and $f(x+y) = f(x)f(y)$.

(7) Explain why $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \dfrac{x^5}{5!} + \dfrac{x^6}{6!} + \cdots$.

(8) Explain why $\ln x$ is the inverse function to e^x.

(9) Verify the identity $e^{x+y} = e^x e^y$.

(10) Verify the identity $e^{-x} = \dfrac{1}{e^x}$.

(11) Verify the identity $(e^x)^n = e^{nx}$.

(12) Verify the identity $e^0 = 1$.

(13) Verify the identity $\ln(xy) = \ln x + \ln y$.

(14) Verify the identity $-\ln x = \ln(1/x)$.

(15) Verify the identity $\ln x^n = n \ln x$.

(16) Verify the identity $\ln 1 = 0$.

(17) Verify the identity $e^{ix} = \cos x + i \sin x$.

(18) Verify the identity $\cos^2 x + \sin^2 x = 1$.

(19) Verify the identity $\sin(x+y) = \sin x \cos y + \cos x \sin y$.

(20) Verify the identity $\cos(x+y) = \cos x \cos y - \sin x \sin y$.

(21) Explain why $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$.

(22) Explain why $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$.

(23) Verify the identity $\sin(-x) = -\sin x$.

(24) Verify the identity $\cos(-x) = \cos x$.

(25) Verify the identity $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$.

(26) Verify the identity $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$.

Problem B. Inverse trigonometric functions.

(1) What is $\sin^{-1} x$?

(2) What is $\cos^{-1} x$?

(3) What is $\tan^{-1} x$?

(4) What is $\cot^{-1} x$?

(5) What is $\sec^{-1} x$?

(6) What is $\csc^{-1} x$?

(7) Verify the identity $\cos(\tan^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$.

(8) Verify the identity $\sin(\tan^{-1} x) = \dfrac{x}{\sqrt{1+x^2}}$.

(9) Verify the identity $\sin(\cos^{-1} x) = \sqrt{1-x^2}$.

(10) Verify the identity $\tan(\cos^{-1} x) = \dfrac{\sqrt{1-x^2}}{x}$.

(11) Verify the identity $\cos(\sin^{-1} x) = \sqrt{1-x^2}$.

(12) Verify the identity $\tan(\cot^{-1} x) = 1/x$.

(13) Verify the identity $\cot(\cot^{-1} 2) = 2$.

(14) Verify the identity $\sin(\cot^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$.

(15) Verify the identity $\cos(\cot^{-1} x) = \dfrac{x}{\sqrt{1+x^2}}$.

(16) Verify the identity $\sin^{-1}(-x) = -\sin^{-1} x$.

(17) Verify the identity $\tan^{-1}(-x) = -\tan^{-1} x$.

(18) Verify the identity $\tan^{-1} x = \cot^{-1}(1/x)$.

(19) Verify the identity $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$.

(20) Verify the identity $\sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$.

Problem C. Derivatives of the basic functions.

(1) Explain why $\dfrac{de^x}{dx} = e^x$.

(2) Explain why $\dfrac{d\sin x}{dx} = \cos x$.

(3) Explain why $\dfrac{d\cos x}{dx} = -\sin x$.

(4) Explain why $\dfrac{d\tan x}{dx} = \sec^2 x$.

(5) Explain why $\dfrac{d\cot x}{dx} = -\csc^2 x$.

(6) Explain why $\dfrac{d\sec x}{dx} = \tan x \sec x$.

(7) Explain why $\dfrac{d\csc x}{dx} = -\cot x \csc x$.

(8) Explain why $\dfrac{d \ln x}{dx} = \dfrac{1}{x}$.

(9) Explain why $\dfrac{d \sin^{-1} x}{dx} = \dfrac{1}{\sqrt{1-x^2}}$.

(10) Explain why $\dfrac{d \cos^{-1} x}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$.

(11) Explain why $\dfrac{d \tan^{-1} x}{dx} = \dfrac{1}{1+x^2}$.

(12) Explain why $\dfrac{d \cot^{-1} x}{dx} = -\dfrac{1}{1+x^2}$.

(13) Explain why $\dfrac{d \csc^{-1} x}{dx} = -\dfrac{1}{|x|\sqrt{x^2-1}}$.

Problem D. Derivatives with trigonometric functions.

(1) Find $\dfrac{dy}{dx}$ when $y = \sin(3x+2)$.

(2) Find $\dfrac{dy}{dx}$ when $y = \sqrt{\sin x^4}$.

(3) Find $\dfrac{dy}{dx}$ when $y = x^2 \sin x$.

(4) Find $\dfrac{dy}{dx}$ when $y = x \cos x - \sin x$.

(5) Find $\dfrac{dy}{dx}$ when $y = \cos^3 3x$.

(6) Find $\dfrac{dy}{dx}$ when $y = (x^2 + \cos x)^4$.

(7) Find $\dfrac{dy}{dx}$ when $y = \sin x \sin 2x$.

(8) Find $\dfrac{dy}{dx}$ when $y = \dfrac{\sin 2x}{x^2}$.

(9) Find $\dfrac{dy}{dx}$ when $y = \tan x \sin 2x$.

(10) Find $\dfrac{dy}{dx}$ when $y = \sin x^2 - \dfrac{\tan x}{1+x^2}$.

(11) Find $\dfrac{dy}{dx}$ when $y = \dfrac{2\cos x - x}{x+2}$.

(12) Find $\dfrac{dy}{dx}$ when $y = (1+x^2) + \dfrac{x}{\sin x}$.

(13) Find $\dfrac{dy}{dx}$ when $y = \dfrac{\sin 2x}{\cos x}$.

(14) Find $\dfrac{dy}{dx}$ when $y = \sin(x/3)\csc(2x/3)$.

(15) Find $\dfrac{dy}{dx}$ when $y = \sin(\sin x + \cos x)$.

(16) Find $\dfrac{dy}{dx}$ when $y = \sqrt{\sec^2 x + \csc^2 x}$.

(17) Find $\dfrac{dy}{dx}$ when $y = (x^2-1)\left(\cot x + \dfrac{\tan x}{1+x^2}\right)$.

(18) Find $\dfrac{dy}{dx}$ when $y = \sqrt{\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}}$.

(19) Find $\dfrac{dy}{dx}$ when $y = \dfrac{\sec x + \tan x}{\sec x - \tan x}$.

(20) Find $\dfrac{dy}{dx}$ when $y = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$.

(21) Find $\dfrac{dy}{dx}$ when $y = x^3\tan^2(x/2)$.

(22) If $y = \tan(\cos(\sin\theta))$ find dy/dx.

Problem E. Derivatives with exponentials and logs.

(1) Find $\dfrac{dy}{dx}$ when $y = \left(ex^2 + \dfrac{\pi}{x^3} + x^{7/2}\right)$.

(2) Find $\dfrac{dy}{dx}$ when $y = a^{ax+b}$.

(3) Find $\dfrac{dy}{dx}$ when $y = a^{x^3}$.

(4) Find $\dfrac{dy}{dx}$ when $y = 6^{2x}$.

(5) Find $\dfrac{dy}{dx}$ when $y = \ln(ax^2 + b)$.

(6) Find $\dfrac{dy}{dx}$ when $y = e^{3 \ln x}$.

(7) Find $\dfrac{dy}{dx}$ when $y = e^{2x} - e^{-2x}$.

(8) Find $\dfrac{dy}{dx}$ when $y = e^{x^2+2x}$.

(9) Find $\dfrac{dy}{dx}$ when $y = a^x x^a$.

(10) Find $\dfrac{dy}{dx}$ when $y = xe^x$.

(11) Find $\dfrac{dy}{dx}$ when $y = \ln\left(x + \sqrt{x^2 + a^2}\right)$.

(12) Find $\dfrac{dy}{dx}$ when $y = \dfrac{1 + e^x}{1 - e^x}$.

(13) Find $\dfrac{dy}{dx}$ when $y = \ln\left(\dfrac{x^2 + x + 1}{x^2 - x - 1}\right)$.

(14) Find $\dfrac{dy}{dx}$ if $y = \ln\left[e^x \left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$.

(15) Find $\dfrac{dy}{dx}$ when $y = \ln \ln \ln x^4$.