# MATH 221: Calculus and Analytic Geometry
## Prof. Ram, Fall 2004

## HOMEWORK 5
### DUE October 11, 2004

**Problem A. Evaluating limits when $x \to 0$.**

(1) Evaluate $\lim\limits_{x \to 0} (x^2 - 2)^2 + 6$.

(2) Evaluate $\lim\limits_{x \to 0} \dfrac{5x}{x}$.

(3) Evaluate $\lim\limits_{x \to 0} \dfrac{17x}{2x}$.

(4) Evaluate $\lim\limits_{x \to 0} \dfrac{-317x}{422x}$.

(5) Evaluate $\lim\limits_{x \to 0} \dfrac{-317x - 3}{422x + 5}$.

(6) Evaluate $\lim\limits_{h \to 0} \dfrac{\sqrt{x + h} - \sqrt{x}}{h}$.

(7) Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x + x^2} - 1}{x}$.

(8) Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$.

(9) Evaluate $\lim\limits_{h \to 0} \dfrac{1}{h} \left( \dfrac{1}{\sqrt{x + h}} - \dfrac{1}{\sqrt{x}} \right)$.

(10) Evaluate $\lim\limits_{x \to 0} \dfrac{2x}{\sqrt{a + x} - \sqrt{a - x}}$.

(11) Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$.

(12) Evaluate $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 + x} - 1}$.

(13) Evaluate $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x} - 2}{x^2}$.

(14) Calculate $\lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \sqrt{ax + b}$.

(15) Calculate $\lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = (mx + c)^n$.

**Problem B. Evaluating limits when $x \to a$.**

(1) Evaluate $\lim\limits_{x \to 1} (6x^2 - 4x + 3)$.

(2) Evaluate $\lim\limits_{x \to 7} \dfrac{x^2 - 49}{x - 7}$.

(3) Evaluate $\lim\limits_{x \to 2} \dfrac{x^2 - 6x + 8}{x - 2}$.

(4) Evaluate $\lim\limits_{x \to -5} \dfrac{2x^2 + 9x - 5}{x + 5}$.

(5) Evaluate $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x - 1}$.

(6) Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3}$.

(7) Evaluate $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x + 2}$.

(8) Evaluate $\lim\limits_{x \to 3} \dfrac{x^4 - 81}{x - 3}$.

(9) Evaluate $\lim\limits_{x \to 5} \dfrac{x^5 - 3125}{x - 5}$.

(10) Evaluate $\lim\limits_{x \to a} \dfrac{x^{12} - a^{12}}{x - a}$.

(11) Evaluate $\lim\limits_{x \to a} \dfrac{x^{5/2} - a^{5/2}}{x - a}$.

(12) Evaluate $\lim\limits_{x \to a} \dfrac{(x + 2)^{5/3} - (a + 2)^{5/3}}{x - a}$.

(13) Evaluate $\lim\limits_{x \to 4} \dfrac{x^3 - 64}{x^2 - 16}$.

(14) Evaluate $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^3 - 8}$.

(15) Evaluate $\lim\limits_{x \to 1} \dfrac{x^n - 1}{x - 1}$.

(16) Evaluate $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

(17) Evaluate $\lim\limits_{x \to 2} \dfrac{\sqrt{3 - x} - 1}{2 - x}$.

(18) Evaluate $\lim\limits_{x \to a} \dfrac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$.

(19) Evaluate $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a}$.

**Problem C. Evaluating limits as $x \to \infty$.**

(1) Evaluate $\lim\limits_{x \to \infty} \dfrac{x + 2}{x - 2}$.

(2) Evaluate $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x - 5}{5x^2 + 3x + 1}$.

(3) Evaluate $\lim\limits_{x \to \infty} \dfrac{x^2 - 7x + 11}{3x^2 + 10}$.

(4) Evaluate $\lim\limits_{x \to \infty} \dfrac{2x^3 - 5x + 7}{7x^3 + 2x^2 - 6}$.

(5) Evaluate $\lim\limits_{x \to \infty} \dfrac{(3x - 1)(4x - 5)}{(x + 6)(x - 3)}$.

(6) Evaluate $\lim\limits_{n \to \infty} \dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \cdots + \dfrac{1}{3^n}$.

(7) Evaluate $\lim\limits_{x \to \infty} \dfrac{x}{\sqrt{4x^2 + 1} - 1}$.

(8) Evaluate $\lim\limits_{x \to -\infty} 2^x$.

(9) Evaluate $\lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n$.

(10) Evaluate $\displaystyle\lim_{t\to\infty} \frac{t+1}{t^2+1}$.

(11) Evaluate $\displaystyle\lim_{n\to\infty} \sqrt{n^2+1} - n$.

(12) Evaluate $\displaystyle\lim_{n\to\infty} \sqrt{n^2+n} - n$.

## Problem D. Limits with exponential and log functions.

(1) Evaluate $\displaystyle\lim_{x\to0} \frac{e^x-1}{x}$.

(2) Evaluate $\displaystyle\lim_{x\to0} \frac{a^x-1}{x}$.

(3) Evaluate $\displaystyle\lim_{x\to0} \frac{\ln(1+x)}{x}$.

(4) Evaluate $\displaystyle\lim_{x\to0} (1+x)^{1/x}$.

(5) Evaluate $\displaystyle\lim_{x\to0} \frac{a^x-b^x}{x}$.

(6) Evaluate $\displaystyle\lim_{x\to0} \frac{e^x+e^{-x}-2}{x^2}$.

(7) Evaluate $\displaystyle\lim_{x\to-\infty} 2^x$.

(8) Explain why $\displaystyle\lim_{x\to-1} \ln x$ does not exist.

(9) Explain why $\displaystyle\lim_{x\to0} 2^{1/x}$ does not exist.

(10) Explain why $\displaystyle\lim_{x\to1} 2^{1/(x-1)}$ does not exist.

(11) Calculate $\displaystyle\lim_{\Delta x\to0} \frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = e^{\sqrt{x}}$.

(12) Calculate $\displaystyle\lim_{\Delta x\to0} \frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = \ln(ax+b)$.

(13) Calculate $\displaystyle\lim_{\Delta x\to0} \frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = x^x$.

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## Problem E. Limits with trigonometric functions.

(1) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{4x}$.

(2) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin x \cos x}{3x}$.

(3) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$.

(4) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin^2 x}$.

(5) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan ax}{\tan bx}$.

(6) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin(x/4)}{x}$.

(7) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin mx}{\tan nx}$.

(8) Evaluate $\lim\limits_{\theta \to 0} \dfrac{1 - \cos 6\theta}{\theta}$.

(9) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{3 \tan^2 x}$.

(10) Evaluate $\lim\limits_{x \to 0} \dfrac{\cos^2 x}{1 - \sin x}$.

(11) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan 2x - x}{3x - \sin x}$.

(12) Evaluate $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a}$.

(13) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x - \sin 3x}{\sin x}$.

(14) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan 3x - 2x}{3x - \sin^2 x}$.

(15) Evaluate $\lim\limits_{x \to 0} \dfrac{x^2 - \tan 2x}{\tan x}$.

(16) Evaluate $\displaystyle\lim_{x\to\pi/4}\frac{1-\tan x}{x-\pi/4}$.

(17) Evaluate $\displaystyle\lim_{x\to0}\frac{\tan(x/2)}{3x}$.

(18) Evaluate $\displaystyle\lim_{x\to0}\frac{1-\cos 2x+\tan^2 x}{x\sin x}$.

(19) If $\displaystyle\lim_{x\to0} kx\csc x=\lim_{x\to0} x\csc kx$, explain why $k=\pm1$.

(20) Evaluate $\displaystyle\lim_{h\to0}\frac{\sin(a+h)-\sin a}{h}$.

(21) Evaluate $\displaystyle\lim_{h\to\infty}\frac{\cos(\pi/h)}{h-2}$.

**Problem F. Calculating $\displaystyle\lim_{\Delta x\to0}\frac{\Delta f}{\Delta x}$.**

(1) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=\sin 2x$.

(2) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=\cos 2x$.

(3) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=\sin x^2$.

(4) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=\cos x^2$.

(5) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=\sqrt{\sin x}$.

(6) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=x\sin x$.

(7) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=e^{\sin x}$.

(8) Calculate $\displaystyle\lim_{\Delta x\to0}\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x)=e^{\cos x}$.

(9) Calculate $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \dfrac{\sin x}{x}$.

**Problem G. Limits with inverse trig functions.**

(1) Evaluate $\displaystyle\lim_{x \to 1} \frac{1 - x}{(\cos^{-1} x)^2}$.

(2) Evaluate $\displaystyle\lim_{x \to 1/\sqrt{2}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$.

(3) Evaluate $\displaystyle\lim_{x \to 0} \frac{x(1 - \sqrt{1 - x^2})}{(\sin^{-1} x)^3 \sqrt{1 - x^2}}$.

(4) Evaluate $\displaystyle\lim_{x \to 1} \frac{1 - x}{\pi - 2\sin^{-1} x}$.

(5) Evaluate $\displaystyle\lim_{x \to 0} \frac{\tan^{-1} 2x}{\sin 3x}$.