MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 6
DUE October 18, 2004

Problem A. Graphs of the basic functions.

(1) Graph $f(x) = |x|$.

(2) Graph $f(x) = \lfloor x \rfloor$.

(3) Graph $f(x) = 2$.

(4) Graph $f(x) = x$.

(5) Graph $f(x) = x^2$.

(6) Graph $f(x) = x^3$.

(7) Graph $f(x) = x^4$.

(8) Graph $f(x) = x^5$.

(9) Graph $f(x) = x^6$.

(10) Graph $f(x) = x^{100}$.

(11) Graph $f(x) = x^{-1}$.

(12) Graph $f(x) = x^{-2}$.

(13) Graph $f(x) = x^{-3}$.

(14) Graph $f(x) = x^{-4}$.

(15) Graph $f(x) = x^{-100}$.

(16) Graph $f(x) = e^x$.

(17) Graph $f(x) = \sin x$.

(18) Graph $f(x) = \cos x$.

(19) Graph $f(x) = \tan x$.

(20) Graph $f(x) = \cot x$.

(21) Graph $f(x) = \sec x$.

(22) Graph $f(x) = \csc x$.

(23) Graph $f(x) = \sqrt{x}$.

(24) Graph $f(x) = x^{1/3}$.

(25) Graph $f(x) = x^{1/4}$.

(26) Graph $f(x) = x^{1/5}$.

(27) Graph $f(x) = x^{1/6}$.

(28) Graph $f(x) = \dfrac{1}{\sqrt{x}}$.

(29) Graph $f(x) = x^{-1/3}$.

(30) Graph $f(x) = x^{-1/4}$.

(31) Graph $f(x) = \ln x$.

(32) Graph $f(x) = \sin^{-1} x$.

(33) Graph $f(x) = \cos^{-1} x$.

(34) Graph $f(x) = \tan^{-1} x$.

(35) Graph $f(x) = \cot^{-1} x$.

(36) Graph $f(x) = \sec^{-1} x$.

(37) Graph $f(x) = \csc^{-1} x$.

Problem B. Where is a function continuous?

(1) For which values of x is the function $f(x) = x^2 + 3x + 4$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(2) For which values of x is the function $f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(3) For which values of x is the function $f(x) = \begin{cases} \dfrac{\sin 3x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(4) For which values of x is the function $f(x) = \begin{cases} \dfrac{1 - \cos x}{x^2}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(5) Determine the value of k for which the function $f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0, \end{cases}$ is continuous at $x = 0$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(6) For which values of x is the function $f(x) = \begin{cases} x - 1, & \text{if } 1 \leq x < 2, \\ 2x - 3, & \text{if } 2 \leq x \leq 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(7) For which values of x is the function $f(x) = \begin{cases} \cos x, & \text{if } x \geq 0, \\ -\cos x, & \text{if } x < 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(8) For which values of x is the function $f(x) = \begin{cases} \sin(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(9) Find the value of a for which the function $f(x) = \begin{cases} ax + 5, & \text{if } x \leq 2, \\ x - 1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(10) For which values of x is the function $f(x) = \begin{cases} 1 + x^2, & \text{if } 0 \leq x \leq 1, \\ 2 - x, & \text{if } x > 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(11) For which values of x is the function $f(x) = 2x - |x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(12) Find the value of a for which the function $f(x) = \begin{cases} 2x-1, & \text{if } x < 2, \\ a, & \text{if } x = 2, \\ x+1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(13) For which values of x is the function $f(x) = \begin{cases} \dfrac{|x-a|}{x-a}, & \text{if } x \neq a, \\ 1, & \text{if } x = a, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(14) For which values of x is the function $f(x) = \begin{cases} \dfrac{x-|x|}{2}, & \text{if } x \neq 0, \\ 2, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(15) For which values of x is the function $f(x) = \begin{cases} \sin x, & \text{if } x < 0, \\ x, & \text{if } x \geq 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(16) For which values of x is the function $f(x) = \begin{cases} \dfrac{x^n-1}{x-1}, & \text{if } x \neq 1, \\ n, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(17) Explain how you know that $f(x) = \sec x$ is continuous for all values of x. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(18) For which values of x is the function $f(x) = \cos|x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(19) For which values of x is the function $f(x) = \lfloor x \rfloor$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(20) For which values of x is the function $f(x) = \begin{cases} x^3 - x^2 + 2x - 2, & \text{if } x \neq 1, \\ 4, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(21) For which values of x is the function $f(x) = |x| + |x-1|$, $-1 \leq x \leq 2$, continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Problem C. Existence of limits.

(1) Explain why $\lim_{x\to 0} 1/x$ does not exist.

(2) Explain why $\lim_{x\to \pi/2} \tan x$ does not exist.

(3) Explain why $\lim_{x\to \pi/2} \sec x$ does not exist.

(4) Explain why $\lim_{x\to 0} \csc x$ does not exist.

(5) Explain why $\lim_{x\to -1} \ln x$ does not exist.

(6) Explain why $\lim_{x\to 0} \sin(1/x)$ does not exist.

(7) Explain why $\lim_{x\to \infty} \cos x$ does not exist.

(8) Let $\operatorname{sgn}(x)$ be the sign function. This function is given by $\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases}$
Explain why $\lim_{x\to \infty} \operatorname{sgn}(x)$ does not exist.

(9) Explain why $\lim_{x\to 0} 2^{1/x}$ does not exist.

(10) Explain why $\lim_{x\to 1} 2^{1/(x-1)}$ does not exist.

Problem D. Increasing, decreasing, and concavity.

(1) What does it mean for a function $f(x)$ to be continuous at $x = a$? Explain how to test if a function is continuous at $x = a$.

(2) What does it mean for a function $f(x)$ to be differentiable at $x = a$? Explain how to test if a function is differentiable at $x = a$.

(3) What does $\left.\dfrac{df}{dx}\right|_{x=a}$ indicate you about the graph of $y = f(x)$? Explain why this is true.

(4) What does it mean for a function to be increasing? Explain how to use calculus to tell if a function is increasing. Explain why this works.

(5) What does it mean for a function to be concave up? Explain how to use calculus to tell if a function is concave up. Explain why this works.

(6) What is a critical point? Explain how to find critical points of a function $f(x)$?

(7) What is a point of inflection? Explain how to find points of inflection of a function $f(x)$?

(8) What is an asymptote of a function $f(x)$? Explain how to justify that a given line is an asymptote of $f(x)$?

(9) If $f(x) = |x|$ what is $\left.\frac{df}{dx}\right|_{x=2}$?

(10) Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1, \\ bx + 2, & \text{if } x > 1, \end{cases}$ is differentiable for all values of x.

Problem E. Graphing polynomials.

For each of the following graphing problems also determine

(a) where $f(x)$ is defined,
(b) where $f(x)$ is continuous,
(c) where $f(x)$ is differentiable,
(d) where $f(x)$ is increasing and where it is decreasing,
(e) where $f(x)$ is concave up and where it is concave down,
(f) what the critical points of $f(x)$ are,
(g) where the points of inflection are, and
(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

(1) Graph $f(x) = a$, where a is a constant.

(2) Graph $f(x) = ax + b$, where a and b are constants.

(3) Graph $f(x) = a(x - c) + b$, where a, b and c are constants.

(4) Graph $f(x) = \begin{cases} 2 - x, & \text{if } x \geq 1, \\ x, & \text{if } 0 \leq x \leq 1. \end{cases}$

(5) Graph $f(x) = \begin{cases} 2+x, & \text{if } x \geq 0, \\ 2-x, & \text{if } x < 0. \end{cases}$

(6) Graph $f(x) = \begin{cases} 1-x, & \text{if } x < 1, \\ x^2-1, & \text{if } x \geq 1. \end{cases}$

(7) Graph $f(x) = 2x - x^2$.

(8) Graph $f(x) = x - x^2 - 27$.

(9) Graph $f(x) = 3x^2 - 2x - 1$.

(10) Graph $f(x) = x^3$.

(11) Graph $f(x) = x^3 - x + 1$.

(12) Graph $f(x) = x^3 - x - 1$.

(13) Graph $f(x) = (x-2)^2(x-1)$.

(14) Graph $f(x) = 2x^3 - 21x^2 + 36x - 20$.

(15) Graph $f(x) = 2x^3 + x^2 - 20x$.

(16) Graph $f(x) = 1 - x^4$.

(17) Graph $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

(18) Graph $f(x) = 3x^4 - 16x^3 + 18x^2$.

(19) Graph $f(x) = x^5 - 4x^4 + 4x^3$.

(20) Graph $f(x) = x^3(x-2)^2$.

(21) Graph $f(x) = (x-2)^4(x+1)^3(x-1)$.