MATH 221: Calculus and Analytic Geometry
Prof. Ram, Spring 2000

Lecture 4: MIDTERM EXAM 2
October 20, 2000

This is a 50 minute exam. No books, notes or calculators are allowed. There are 10 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: _____

TA and Section: _____

Problem	Score
1.	
2.	
3.	
4.	
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6.	
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9.	
10.	
Total	

Problem 1. Evaluate $\lim_{x \to 0} \dfrac{\sin 5x - \sin 3x}{\sin x}$.

Problem 2. Graph $f(x) = \lfloor x \rfloor$.

Problem 3. Find $\dfrac{dy}{dx}$ when $y = \sin^{-1} x^3$.

Problem 4. Evaluate $\lim_{x \to 0} \dfrac{a^x - b^x}{x}$.

Problem 5. Let $y = x^2 \sin x + 2x \cos x - 2x$. Find $\dfrac{dy}{dx}$ at $x = 0$ and $x = \pi/2$.

Problem 6. Graph $f(x) = \cos^{-1} x$.

Problem 7. Graph $y = f(x)$ when $x = \cos 2\theta$ and $y = \cos\theta$. Also determine

(a) where $f(x)$ is defined,

(b) where $f(x)$ is continuous,

(c) where $f(x)$ is differentiable,

(d) where $f(x)$ is increasing and where it is decreasing,

(e) where $f(x)$ is concave up and where it is concave down,

(f) what the critical points of $f(x)$ are,

(g) where the points of inflection are, and

(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 8. For which values of x is the function $f(x) = \begin{cases} x^3 - x^2 + 2x - 2, & \text{if } x \neq 1, \\ 4, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Problem 9. If $y = e^{-x}\cos x$ prove that $\dfrac{d^4y}{dx^4} + 4y = 0$.

Problem 10. Evaluate $\lim_{x \to 0} \dfrac{\sin mx}{\tan nx}$.