MATH 221: Calculus and Analytic Geometry
Prof. Ram, Spring 2000

Lecture 3: MIDTERM EXAM 2
October 20, 2000

This is a 50 minute exam. No books, notes or calculators are allowed. There are 11 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: ______________________________

TA and Section: ______________________________

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
Total	

Problem 1. Explain how you know that $f(x) = \sec x$ is continuous for all values of x. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Problem 2. Explain why $\lim_{x \to 1} 2^{1/(x-1)}$ does not exist.

Problem 3. Let $y = \cos(\sin x^2)$. Find $\left.\dfrac{dy}{dx}\right|_{x=\pi/3}$.

Problem 4. Find $\dfrac{dy}{dx}$ when $y = \tan^{-1} 5x$.

Problem 5. Give a series expansion for $\dfrac{1}{1-x}$.

Problem 6. Evaluate $\lim_{x \to 1} (6x^2 - 4x + 3)$.

Problem 7. For which values of x is the function $f(x) = \cos|x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Problem 8. Evaluate $\lim_{x \to 0} \dfrac{\sin x \cos x}{3x}$.

Problem 9. Find $\dfrac{dy}{dx}$ when $y = \cot^{-1}\left(\dfrac{1+\cos 3x}{1-\cos 3x}\right)^{1/2}$.

Problem 10. Graph $f(x) = x - x^2 - 27$. Also determine

(a) where $f(x)$ is defined,

(b) where $f(x)$ is continuous,

(c) where $f(x)$ is differentiable,

(d) where $f(x)$ is increasing and where it is decreasing,

(e) where $f(x)$ is concave up and where it is concave down,

(f) what the critical points of $f(x)$ are,

(g) where the points of inflection are, and

(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 11. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \sqrt{ax+b}$.

Fully evaluate the limit; do not short cut with "Formula 3". Assume that a and b are constants.