MATH 221: Calculus and Analytic Geometry
Prof. Ram, Spring 2000

SAMPLE FINAL EXAM 1
December 2, 2000

This is a 2 hour exam. No books, notes or calculators are allowed. There are 16 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: ______________________________

TA and Section: ______________________________

Problem	Score
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Problem 1. Compute $\dfrac{6-i}{4+2i}$ and graph the result.

Problem 2. Find $\dfrac{dy}{dx}$ when $y = \left(\dfrac{x-3}{x-4}\right)^2$.

Problem 3. Find the third derivative of $x^2 \ln x$ with respect to x.

Problem 4. Find $\dfrac{dy}{dx}$ when $y = \sec^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$.

Problem 5. Evaluate $\lim_{x \to 0} \dfrac{-317x}{422x}$.

Problem 6. Graph $f(x) = x^{-2}$.

Problem 7. Graph $y = f(x)$ when $x^2 - y^2 = 1$. Also determine

(a) where $f(x)$ is defined,
(b) where $f(x)$ is continuous,
(c) where $f(x)$ is differentiable,
(d) where $f(x)$ is increasing and where it is decreasing,
(e) where $f(x)$ is concave up and where it is concave down,
(f) what the critical points of $f(x)$ are,
(g) where the points of inflection are, and
(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 8. Given the perimeter of a rectangle show that its diagonal is minimum when it is a square. Make up a word problem for which this gives the solution.

Problem 9. $\displaystyle\int \frac{1}{1-\cos 2x}\, dx$

Problem 10. $\int_1^3 \left(\frac{1}{t^2} - \frac{1}{t^4} \right) \, dt$

Problem 11. Find the volume generated when the area bounded by $y = \sin x$, $0 \leq x \leq \pi$, and $y = 0$ is rotated about the x-axis.

Problem 12. Graph $f(x) = \sin x$, $0 \le x \le \pi/2$, and find its average value. Indicate the average value on the graph. Draw a rectangle with base $0 \le x \le \pi/2$ and with area equal to the area under the graph of $f(x)$.

Problem 13. A sample contains 4.6 mg of $^{131}_{53}\mathrm{I}$. How many mg will remain after 3.0 days? The half life of $^{131}_{53}\mathrm{I}$ is 8.0 days.

Problem 14. Find $\dfrac{dy}{dx}$ when $y = \sin^{-1} x + \sin^{-1}\sqrt{1 - x^2}$.

Problem 15. The bottom of a rectangular swimming pool is 25×40 meters. Water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of the water in the tank is rising.

Problem 16. Suppose that $f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + \cdots$. Show that $c_3 = \dfrac{1}{3!}\left(\left.\dfrac{d^3 f}{dx^3}\right|_{x=a}\right)$.