MATH 221: Calculus and Analytic Geometry
Prof. Ram, Spring 2000

SAMPLE FINAL EXAM 2
December 4, 2000

This is a 2 hour exam. No books, notes or calculators are allowed. There are 16 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: ______________________________

TA and Section: ______________________________

Problem	Score
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Problem 1. Verify the identity $(1/2)\sin 2A = \dfrac{\tan A}{1+\tan^2 A}$.

Problem 2. Find $\frac{dy}{dx}$ when $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$.

Problem 3. Find $\dfrac{dy}{dx}$ when $y = \sin x \sin 2x$.

Problem 4. Find $\frac{dy}{dx}$ when $y = \frac{\sqrt{x + \ln \tan x}}{xe^{2x}}$.

Problem 5. Evaluate $\lim_{x\to 0} \dfrac{2x}{\sqrt{a+x}-\sqrt{a-x}}$.

Problem 6. Graph $f(x) = \tan^{-1} x$.

Problem 7. Graph $f(x) = x + 1/x$. Also determine

(a) where $f(x)$ is defined,

(b) where $f(x)$ is continuous,

(c) where $f(x)$ is differentiable,

(d) where $f(x)$ is increasing and where it is decreasing,

(e) where $f(x)$ is concave up and where it is concave down,

(f) what the critical points of $f(x)$ are,

(g) where the points of inflection are, and

(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 8. Find the equation of the tangent to the curve $x^2 + 2y = 8$ which is perpendicular to the line $x - 2y + 1 = 0$.

Problem 9. $\displaystyle\int \frac{2x-1}{\sqrt{x^2-x-1}}\,dx$

Problem 10. $\int_{-e^2}^{-e} \frac{3}{x}\ dx$

Problem 11. Find the volume generated when the area bounded by $y = x^2 - 2x$ and $y = 0$ is rotated about the x-axis.

Problem 12. Find the length of the curve $y = (1/3)x^3 + 1/4x$ from $x = 1$ to $x = 3$.

Problem 13. Give three examples which illustrate that a limit problem that looks like it is coming out to 0^0 could be really getting closer and closer to almost anything and must be looked at a different way.

Problem 14. $\int x^2 e^{x^3} \cos e^{x^3} \; dx$

Problem 15. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

Problem 16. Explain why $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$.