Math 521 Lecture 3: Homework 1

Fall 2004, Professor Ram

Due September 13, 2004

1 Vocabulary

Define the following terms.

1. positive integers
2. nonnegative integers
3. integers
4. rational numbers
5. real numbers
6. irrational numbers
7. complex numbers
8. algebraic numbers
9. transcendental numbers
10. set
11. element
12. emptyset
13. subset
14. proper subset
15. equal (sets)
16. function
17. well defined
18. equal (functions)
19. image
20. inverse image

21. fiber
22. injective
23. surjective
24. bijective
25. composition (of functions)
26. identity (function)
27. inverse function
28. union
29. intersection
30. disjoint
31. product
32. index
33. cardinality
34. finite
35. countable
36. infinite
37. uncountable

2 Exercises

1. Give an example of a noncommutative operation.
2. Give an example of a nonassociative operation.
3. Show that there is a unique decimal expansion of 0 but that there are two decimal expansions of 1.
4. Prove Theorem 1.1 in section 1 of the Lecture Notes.
5. Prove Theorem 1.2 in section 1 of the Lecture Notes.
6. Prove Theorem 1.3 in section 1 of the Lecture Notes.
7. Prove that $\mathbb{Q} \subseteq \overline{\mathbb{Q}}$.
8. Prove that π is irrational.
9. Prove that π is transcendental.
10. Prove that e is irrational.

11. Prove that e is transcendental.
12. Prove that if $a \in \mathbb{Q}$ then e^a is transcendental.
13. Show that if $x + y = x + z$ then $y = z$ and that if $xy = xz$ then $y = z$. (Correct the problem as necessary.)
14. Show that additive identity is unique, the multiplicative identity is unique. and the inverse of an element is unique.
15. Show that $-(-5) = 5$, $1/(1/5) = 5$, $(-1)5 = -5$, $0 + 0 = 0$, and $0 \cdot 5 = 0$.
16. Define b^a. What values can b and a take? Is b^a determined by its properties?
17. Define $\log_b a$. What values can b and a take? Is $\log_b a$ determined by its properties?
18. Explain why we use the notation $A \subseteq B$ and not the notation $A \subset B$.
19. Give an example of something that looks like a function that is not a function.
20. Give examples of functions $f\colon \mathbb{R} \to \mathbb{R}$ that are (a) injective but not surjective, (b) surjective but not injective, and (c) bijective but not the identity.
21. Define and graph the constant function $1\colon \mathbb{R} \to \mathbb{R}$ and define and graph the identity function $\mathrm{id}_{\mathbb{R}}$.
22. List your favorite functions and give their definitions.
23. Let $f\colon S \to T$ be a function. Show that the inverse function to f exists if and only if f is bijective.
24. Do Chapter 1, problem 1 in baby Rudin.
25. Do Chapter 1, problem 2 in baby Rudin.
26. Do Chapter 1, problem 10 in baby Rudin.
27. Do Chapter 1, problem 11 in baby Rudin.
28. Do Chapter 1, problem 14 in baby Rudin.
29. Do Chapter 1, problem 16 in baby Rudin.
30. Do Chapter 1, problem 17 in baby Rudin.
31. Do Chapter 1, problem 19 in baby Rudin.
32. Do Chapter 2, problem 1 in baby Rudin.
33. Do Chapter 2, problem 2 in baby Rudin.
34. Do Chapter 2, problem 3 in baby Rudin.