Math 521 Lecture 3: Homework 4
Fall 2004, Professor Ram
Due October 4, 2004

1 Favourites

An arsenal of examples in your head is crucial to processing mathematical concepts. For each of the following, list your favourite examples. Make sure your list includes enough examples to develop an understanding of the concept. If it is not clear that your example is an example then prove that it is.

1. elements of $\mathbb{Q}[x]$
2. elements of $\mathbb{Q}[[x]]$
3. countable sets
4. uncountable sets
5. finite sets

2 Exercises

1. Show that $\mathrm{Card}(\mathbb{Z}_{>0}) = \mathrm{Card}(\mathbb{Z}_{\geq 0})$.
2. Show that $\mathrm{Card}(\mathbb{Z}_{\geq 0}) = \mathrm{Card}(\mathbb{Z})$.
3. Show that $\mathrm{Card}(\mathbb{Z}) = \mathrm{Card}(\mathbb{Q})$.
4. Show that $\mathrm{Card}(\mathbb{Q}) \neq \mathrm{Card}(\mathbb{R})$.
5. Show that $\mathrm{Card}(\mathbb{R}) = \mathrm{Card}(\mathbb{C})$.
6. Show that subsets of countable sets are countable.
7. Show that countable unions of countable sets are countable.
8. Show that if S is countable and $n \in \mathbb{Z}_{>0}$ then S^n is countable.
9. Show that $2^{\mathbb{Z}_{>0}}$ is uncountable.
10. Show that $[a, b]$ is uncountable.
11. Show that $\mathbb{R}$ is uncountable.

12. Show that the Cantor set is uncountable
13. Show that every perfect subset of $\mathbb{R}^k$ is uncountable.
14. Show that the Cantor set is perfect.
15. Do Chapter 1, Problem 3 in baby Rudin.
16. Do Chapter 1, Problem 4 in baby Rudin.
17. Do Chapter 1, Problem 5 in baby Rudin.
18. Do Chapter 1, Problem 6 in baby Rudin.
19. Do Chapter 1, Problem 7 in baby Rudin.
20. Do Chapter 1, Problem 8 in baby Rudin.
21. Do Chapter 1, Problem 9 in baby Rudin.
22. Show that if S is a lattice then the intersection of two intervals is an interval. Give an example to show that this is not necessarily true if S is not a lattice.
23. Show that every well ordered set is totally ordered.
24. Show that there exist totally ordered sets that are not well ordered.
25. What is the axiom of choice?
26. What is Zorn's lemma?
27. Show that the axiom of choice, Zorn's lemma and the statement that every set can be well ordered are all equivalent.
28. Explain how $\mathbb{Z}_{\geq 0}$ is an ordered monoid.
29. Show that there is a unique extension of the ordering on $\mathbb{Z}_{\geq 0}$ to $\mathbb{Z}$ so that $\mathbb{Z}$ is an ordered group.
30. Show that there is a unique extension of the ordering on $\mathbb{Z}$ to an ordering on $\mathbb{Q}$ so that $\mathbb{Q}$ is an ordered field.
31. If $\leq$ is the ordering on $\mathring{A}$ given by the previous problem show that

$$\text{if } x, y \in \mathbb{Q} \text{ then } x \leq y \text{ if and only if } y - x \geq 0.$$

3 Vocabulary

Define the following terms.

1. cardinality
2. finite
3. countable

4. uncountable
5. Cantor set
6. perfect set
7. partial order
8. poset
9. total order
10. well order
11. interval
12. lower order ideal
13. open interval
14. closed interval
15. upper bound
16. lower bound
17. greatest lower bound
18. least upper bound
19. supremum
20. infimum
21. lattice
22. ordered monoid
23. ordered group
24. ordered ring
25. ordered field
26. positive
27. negative
28. absolute value
29. coprime
30. irreducible
31. sgn