

MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 14: SELECTED ANSWERS

Problem A. Derivatives with all functions mixed together.

- (1) $\frac{dy}{dx} = \frac{2(\sec x + \tan x \sin x)}{(\tan x + \cos x)^3}$.
- (2) $\frac{dy}{dx} = \frac{x \cos x \sin x}{2\sqrt{x} \sin x}$.
- (3) $\frac{dy}{dx} = \frac{\cos 3x(1 + 2 \cos 2x) + 3 \sin 3x(x + \sin 2x)}{\cos^2 3x}$.
- (4) $\frac{dy}{dx} = 5e^{5x} \ln \sec x + e^{5x} \tan x$.
- (5) $\frac{dy}{dx} = \frac{5x^4 \sin^{-1} 2x - 2x^5(1 - 4x^2)^{-1/2}}{(\sin^{-1} 2x)^2}$.
- (6) $\frac{dy}{dx} = 2x \cos x^2 - \frac{(1 + x^2) \sec^2 x - 2x \tan x}{(1 + x^3)^2}$.
- (7) $\frac{dy}{dx} = 3(\tan \sqrt{x} + x^2 - \sin x)^2 \left(\frac{\sec^2 \sqrt{x}}{2\sqrt{x}} + 2x - \cos x \right)$.
- (8) $\frac{dy}{dx} = \frac{\sin^2 x(2 \cos 3x(3 \cos^2 x - \sec^2 x) + 3 \sin 2x \sin 3x)}{(\cos 3x)^2}$.
- (9) $\frac{dy}{dx} = e^x(\sec^2 x + \tan x) + \frac{\sin x - x \cos x \ln x}{x \sin^2 x}$.
- (10) $\frac{dy}{dx} = 2a^x(1/x + \ln x \ln a)$.
- (11) $\frac{dy}{dx} = \frac{3x^2 - x^{3/2} - 1}{6x\sqrt{x}}$.
- (12) $\frac{dy}{dx} = \frac{x^2 - 4x + 1}{(x^2 - x + 1)^2}$.

$$(13) \frac{dy}{dx} = \frac{\ln \sin x + x \cot x \ln x}{2x\sqrt{1 + \ln x \ln \sin x}}.$$

$$(14) \frac{dy}{dx} = \frac{7}{2}x^{-1/2} - \frac{35}{2}x^{-9/2} + \frac{4x^3}{\sqrt{1-x^3}} + \frac{\csc^2 x}{\cos x}.$$

$$(15) \frac{dy}{dx} = 2 \sin x \cos^4 x - 3 \sin^3 x \cos^2 x.$$

$$(16) \frac{dy}{dx} = m \cos mx \cos nx - n \sin mx \sin nx.$$

$$(17) \frac{dy}{dx} = \sin^{m-1} x \cos^{n-1} x (m \cos^2 x - n \sin^2 x).$$

$$(18) \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

$$(19) \frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}.$$

$$(20) \frac{dy}{dx} = \frac{1}{2x^2} \left((\sqrt{a+x} - \sqrt{a-x})^2 - \frac{2x^2}{\sqrt{a^2-x^2}} \right).$$

$$(21) \frac{dy}{dx} = 96x^3 + 150x^2 + 70x + 10.$$

$$(22) \frac{dy}{dx} = -2 \left(\tan \sqrt{1-x^2} \right) \left(\sec^2 \sqrt{1-x^2} \right) \frac{x}{\sqrt{1-x^2}}.$$

$$(23) \frac{dy}{dx} = \frac{x - \sin 2x}{x^3 \cos^2 x}.$$

$$(24) \frac{dy}{dx} = \frac{e^{2x}(2x \ln x - 1)}{x} (\ln x)^2.$$

$$(25) \frac{dy}{dx} = e^{x^2} (1+x^2)^{-3/2} (x(1+2x^2) \tan^{-1} x + 1).$$

$$(26) \frac{dy}{dx} = \frac{e^{\sqrt{x}+2}}{2\sqrt{x}} - \frac{e^{\sqrt{x}+2}}{2\sqrt{x+2}}.$$

$$(27) \frac{dy}{dx} = 2(\ln 7)(x+1)7^{x^2+2x}.$$

$$(28) \frac{dy}{dx} = -3 \cot^2(e^{3x} x^x) \csc^2(e^{3x} x^x) e^{3x} x^x (4 + \ln x).$$

$$(29) \frac{dy}{dx} = \frac{1}{2(1+x^2)}.$$

$$(30) \frac{dy}{dx} = \frac{1}{(1+x^2)\tan^{-1}x}.$$

$$(31) \frac{dy}{dx} = \frac{2}{1+x^2}.$$

$$(32) \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

$$(33) \frac{dy}{dx} = \frac{2}{1+x^2}.$$

$$(34) \frac{dy}{dx} = 0.$$

$$(35) \frac{dy}{dx} = \frac{-1}{1+x^2}.$$

$$(36) \frac{dy}{dx} = \sin^{-1}x.$$

$$(37) \frac{dy}{dx} = \cos^{-1}2x.$$

$$(38) \frac{dy}{dx} = \frac{1}{5+3\cos x}.$$

$$(39) \frac{dy}{dx} = 1/2.$$

$$(40) \frac{dy}{dx} = \frac{\cos^{-1}x - x\sqrt{1-x^2}}{(1-x^2)^{3/2}}.$$

$$(41) \frac{dy}{dx} = \frac{2(a^2-x^2)-x}{4\sqrt{a^2-x^2}} + \frac{a^2}{2\sqrt{a^2-x^2}}.$$

$$(42) \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

$$(43) \frac{dy}{dx} = \frac{2}{1+x^2}.$$

$$(44) \frac{dy}{dx} = 2x^2 \cos 2x + 2x \sin 2x - \frac{(x+1)\sin x + \cos x}{(x+1)^2}.$$

$$(45) \quad \frac{dy}{dx} = \frac{2x - x^4}{(x^2 + 1)^2} + 2x^4 \cos 2x + 4x^3 \sin 2x.$$

$$(46) \quad \frac{dy}{dx} = (2 \ln x)x^{\ln x - 1}.$$

$$(47) \quad \frac{dy}{dx} = (\tan x)^{\cot x} (\csc^2 x)(1 - \ln \tan x).$$

$$(48) \quad \frac{dy}{dx} = x^{\cos^{-1} x} \left(\frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1 - x^2}} \right) + (\tan x)^{\cot x} (\csc^2 x)(1 - \ln \tan x) \\ + (\cot x)^{\tan x} (\sec^2 x)(\ln \cot x - 1).$$

$$(49) \quad \frac{dy}{dx} = (\cos x)(e^x)(\ln x)(x^x)(x^{\cos^{-1} x}) + (\sin x)(e^x)(\ln x)(x^x)(x^{\cos^{-1} x}) \\ + (\sin x)(e^x)(1/x)(x^x)(x^{\cos^{-1} x}) + (\sin x)(e^x)(\ln x)(1 + \ln x)(x^x)(x^{\cos^{-1} x}) \\ + (\sin x)(e^x)(\ln x)(x^x) + x^{\cos^{-1} x} \left(\frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1 - x^2}} \right).$$

$$(50) \quad \frac{dy}{dx} = \frac{\ln y - (y/x)}{\ln x - (x/y)}.$$

$$(51) \quad \frac{dy}{dx} = - \left(\frac{y}{x} \right)^3.$$

$$(52) \quad \frac{dy}{dx} = -\frac{y}{x}.$$

$$(53) \quad \frac{dy}{dx} = \frac{\cos(x + y) - y}{x - \cos(x + y)}.$$

$$(54) \quad \frac{dy}{dx} = - \left(\frac{b}{a} \right)^m \left(\frac{x}{y} \right)^{m-1}.$$

$$(55) \quad \frac{dy}{dx} = y/x.$$

$$(56) \quad \frac{dy}{dx} = \frac{x - y}{x(1 + \ln x)}.$$

Problem B. Integrals with mixed functions.

$$(1) \quad (1/2)(\sin^{-1} x)^2 + c$$

$$(2) \quad (-1/2) \cos(2 \tan^{-1} x) + c$$

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|---|--|
| (3) $\sin(\ln x) + c$ | (4) $-\cot(\ln x) + c$ |
| (5) $e^{\tan x} + c$ | (6) $-e^{\cos^2 x} + c$ |
| (7) $(1/2)(\ln(\sin x))^2 + c$ | (8) $\ln \ln \sin x + c$ |
| (9) $(1/2)(\ln(\sec x + \tan x))^2 + c$ | (10) $(1/4)(\tan^{-1} x^2)^2 + c$ |
| (11) $(1/4)(\sin^{-1} x^2)^2 + c$ | (12) $\ln \sin^{-1} x + c$ |
| (13) $\ln x + \ln(\sec x) + c$ | (14) $\ln \ln(\tan x) + c$ |
| (15) $\tan(1 + \ln x) + c$ | (16) $(1/2) \cot(2e^{-x} + 5) + c$ |
| (17) $(1/3) \sin(e^{x^3}) + c$ | (18) $\frac{e^{m \tan^{-1} x}}{m} + c$ |
| (19) $\tan(xe^x) + c$ | (20) $2 \sin(e^{\sqrt{x}}) + c$ |

Problem C. Areas of regions.

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|--|---|
| (1) πab sq. units | (2) $15/2$ sq. units |
| (3) 6π sq. units | (4) $2\sqrt{2}/3 + 9\pi/2 - 9 \sin^{-1}(1/3)$ sq. units |
| (5) $\pi/2 + 1/3$ sq. units | (6) $\pi a^2/4 - 2a^2/3$ sq. units |
| (7) $\sqrt{2}/6 + 9\pi/8 - (9/4) \sin^{-1}(1/3)$ sq. units | |
| (8) $\pi/4 - 1/2$ sq. units | (9) $23/6$ sq. units |
| (10) $1/3$ sq. units | |

Problem D. Different types of volume problems.

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| (1) $f(x) = \sqrt{\frac{2x - a}{\pi}}$ | (2) $f(x) = \pm \sqrt{\frac{2x + 1}{\pi}}$ |
| (3) $32\pi/3$ cubic units | (4a) $2\pi a^3$ cubic units |
| (4b) $16\pi a^3/15$ cubic units | (4c) $8\pi a^3/5$ cubic units |

(5) $(8\pi/3)(2 - \sqrt{2})$ cubic units

(6) $(\pi/15)(88\sqrt{2} + 107)$ cubic units

(7) $112\pi a^3/15$ cubic units

(8) hs^2 cubic units

(9) hs^2 cubic units

(10) $8r^3/3$ cubic units

(11) $\pi^2/4$ cubic units

(12) $28\pi/3$ cubic units