MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 4: SELECTED ANSWERS

Problem A. Expansions.

(10) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots.$

(11) $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots.$

(12) $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots.$

(13) $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots.$

(14) $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots.$

(15) $\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots.$

Problem B. Derivatives at a point.

(1) $\left.\dfrac{dy}{dx}\right|_{x=\pi/4}$ is undefined.

(2) $\left.\dfrac{dy}{dx}\right|_{x=0} = 0$ and $\left.\dfrac{dy}{dx}\right|_{x=\pi/2} = -4\left(\dfrac{2+\pi}{4+\pi^2}\right)^2.$

(3) $\left.\dfrac{dy}{dx}\right|_{x=\pi/3} = -\dfrac{2\pi}{3}\sin(\sin(\pi^2/9))\cos(\pi^2/9).$

(4) $\left.\dfrac{dy}{dx}\right|_{x=\pi^2/16} = \dfrac{27}{\pi}.$

(5) $\left.\dfrac{dy}{dx}\right|_{x=0} = 0$ and $\left.\dfrac{dy}{dx}\right|_{x=\sqrt{\pi/2}} = \dfrac{-2\sqrt{\pi}}{(2+\pi)^{3/2}}.$

Problem D. Parametric equations.

(1) $\dfrac{dy}{dx} = (-b/a)\cot\theta.$

(2) $\dfrac{dy}{dx} = \tan(\theta/2).$

(3) $\dfrac{dy}{dx} = \dfrac{3b}{2a}\tan\theta.$

(4) $\dfrac{dy}{dx} = -\cot\phi.$

(5) $\dfrac{-1}{4a}\csc^4(t/2)$ (6) $-1/a$ (7) $-3/2$

Problem E. Implicit differentiation.

(1) $\dfrac{dy}{dx} = \dfrac{(1+x^2)\sin x - (y^2+2x)\cos x - y\sec^2 x}{2y\sin x + \tan x}.$

(2) $\dfrac{dy}{dx} = \dfrac{y(2xy - y^2\cos(xy) - 1)}{xy^2\cos(xy) + y^2 - x}.$

(3) $\dfrac{dy}{dx} = \dfrac{2xy - 2(1+x^2)^2\tan x\sec^2 x}{(1+x^2)^2(4y+\cos y) + (1+x^2)}.$

(4) $\dfrac{dy}{dx} = \dfrac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)}.$

(5) $\dfrac{dy}{dx} = \dfrac{-y(ay^2\cos(xy) - b\sin(x/y))}{x(ay^2\cos(xy) + b\sin(x/y))}.$

(6) $\dfrac{dy}{dx} = \tan^{-1} e^x + \dfrac{xe^x}{1+e^{2x}}.$

Problem F. Derivatives with inverse trig functions.

(1) $\dfrac{dy}{dx} = \dfrac{3x^2}{\sqrt{1-x^6}}.$

(2) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}(1+x)}.$

(3) $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{1-9x^2}}.$

(4) $\dfrac{dy}{dx} = \dfrac{2}{x\sqrt{x^4-1}}.$

(5) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x(1-x)}}$.

(6) $\dfrac{dy}{dx} = \dfrac{-\cot x}{\sqrt{\sin^2 x - 1}}$.

(7) $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{x-1}}$.

(8) $\dfrac{dy}{dx} = \dfrac{\cos(\tan^{-1} x)}{1+x^2}$.

(9) $\dfrac{dy}{dx} = \dfrac{-x}{\sqrt{1-x^2}} + \cos^{-1} x$.

(10) $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1-x^2}} + \sin^{-1} x$.

(11) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}(1+x)} - \dfrac{1}{1+x^2}$.

(12) $\dfrac{dy}{dx} = 2x \tan^{-1} x + 1$.

(13) $\dfrac{dy}{dx} = \dfrac{-\tan x}{\sqrt{1-x^2}} + \cos^{-1} x \sec^2 x$.

(14) $\dfrac{dy}{dx} = \dfrac{2}{1-x^4}$.

(15) $\dfrac{dy}{dx} = -\sqrt{1-x^2} - 2x \cos^{-1} x$.

(16) $\dfrac{dy}{dx} = \dfrac{\tan x}{1+x^2} + \tan^{-1} x \sec^2 x$.

(17) $\dfrac{dy}{dx} = 0$.

(18) $\dfrac{dy}{dx} = \left(\dfrac{-a}{x^2+a^2}\right)(-\tan^{-1}(a/x) + \cot^{-1}(x/a))$.

(19) $\dfrac{dy}{dx} = \dfrac{6(\tan^{-1} 2x)^2}{1+4x^2}$.

(20) $\dfrac{dy}{dx} = \dfrac{2x \sec^2 x}{\sqrt{1 - \tan^2 x^2}}$.

(21) $\dfrac{dy}{dx} = \dfrac{2}{1 + x^2}$.

(22) $\dfrac{dy}{dx} = \dfrac{-2}{1 + x^2}$.

(23) $\dfrac{dy}{dx} = \dfrac{3}{(1 + x^2)^2}$.

(24) $\dfrac{dy}{dx} = \dfrac{2x}{1 + x^4}$.

(25) $\dfrac{dy}{dx} = \dfrac{2}{1 + x^2}$.

(26) $\dfrac{dy}{dx} = 1/2$.

(27) $\dfrac{dy}{dx} = 3/2$.

(28) $\dfrac{dy}{dx} = \dfrac{-(b^2 - a^2)\sin x}{b + a\cos x}$.

(29) $\dfrac{dy}{dx} = \dfrac{\sqrt{3}}{2 + \cos x}$.

(30) $\dfrac{dy}{dx} = 1/2$.

(31) $\dfrac{dy}{dx} = 1$.

Problem G. Derivatives with trig functions.

(1) $\dfrac{dy}{dx} = -x \sin x$.

(2) $\dfrac{dy}{dx} = -9 \cos^2 3x \sin 3x$.

(3) $\dfrac{dy}{dx} = 4(x^2 + \cos x)^3(2x - \sin x)$.

(4) $\dfrac{dy}{dx} = 2 \sin x(3 \cos^2 x - 1)$.

(5) $\dfrac{dy}{dx} = \dfrac{2x\cos 2x - 2\sin 2x}{x^3}$.

Problem H. Derivatives with exponentials and logs.

(1) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2+a^2}}$.

(2) $\dfrac{dy}{dx} = \dfrac{2e^x}{(1-e^x)^2}$.

(3) $\dfrac{dy}{dx} = \dfrac{-2x(x+2)}{(x^2+x+1)(x^2-x-1)}$.

(4) $\dfrac{dy}{dx} = \dfrac{x^2-1}{x^2-4}$.

(5) $\dfrac{dy}{dx} = \dfrac{4}{\ln(\ln x^4)(\ln x^4)x}$.

Problem I. Derivatives with exponentials, logs and trig functions.

(1) $\dfrac{dy}{dx} = 0$.

(2) $\dfrac{dy}{dx} = m\cot x + n\tan x$.

(3) $\dfrac{dy}{dx} = e^{ax}(a\sin bx + b\cos bx)$.

(4) $\dfrac{dy}{dx} = 2\csc x$.

(5) $\dfrac{dy}{dx} = \sec 2x$.

(6) $\dfrac{dy}{dx} = e^{ax}(a\cos(bx+c) - b\sin(bx+c))$.

(7) $\dfrac{dy}{dx} = \dfrac{x(1+2\csc 2x) - 2(x+\ln\tan x)(1+2x)}{2(\sqrt{x+\ln\tan x})(x^2e^{2x})}$.

(8) $\dfrac{dy}{dx} = \dfrac{2(x\cos x + \sin x)}{1-x^2\sin^2 x}$.

(9) $\dfrac{dy}{dx} = \csc x$.

(10) $\dfrac{dy}{dx} = \sec x$.