

MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 6: SELECTED ANSWERS

Problem B. Where is a function continuous?

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|---------------------------|-------------------------|-------------------------|
| (1) all x | (2) all x | (3) $x \neq 0$ |
| (4) $x \neq 0$ | (5) $k = 2/5$ | (6) $1 \leq x \leq 3$ |
| (7) $x \neq 0$ | (8) $x \neq 0$ | (9) $a = -2$ |
| (10) $x \geq 0, x \neq 1$ | (11) all x | (12) $a = 3$ |
| (13) $x \neq a$ | (14) $x \neq 0$ | (15) all x |
| (16) all x | (18) all x | (19) x not an integer |
| (20) $x \neq 1$ | (21) $-1 \leq x \leq 2$ | |

Problem D. Increasing, decreasing, and concavity.

- (9) 1 (10) $a = 3$ and $b = 5$

Problem E. Graphing polynomials.

- (1) Defined for all x ; continuous for all x ; differentiable for all x ; increasing nowhere; decreasing nowhere; concave up nowhere; concave down nowhere; all x are critical points; no points of inflection; $y = a$ is an asymptote.
- (2) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for all x if $a > 0$; decreasing for all x if $a < 0$; concave up nowhere; concave down nowhere; all x are critical points if $a = 0$ and there are no critical points if $a \neq 0$; no points of inflection; $y = ax + b$ is an asymptote.
- (3) Same as (2).
- (4) Defined for $x \geq 0$; continuous for $x \geq 0$; differentiable for $x \neq 1$; increasing for $0 < x < 1$; decreasing for $x > 1$; concave up nowhere; concave down nowhere; critical points at $x = 1$ and $x = 0$; no points of inflection; $y = 2 - x$ is an asymptote as $x \rightarrow \infty$.

- (5) Defined for all x ; continuous for all x ; differentiable for $x \neq 0$; increasing for $x > 0$; decreasing for $x < 0$; concave up nowhere; concave down nowhere; critical point at $x = 0$; no points of inflection; $y = 2 + x$ is an asymptote as $x \rightarrow \infty$, $y = 2 - x$ is an asymptote as $x \rightarrow -\infty$.
- (6) Defined for all x ; continuous for all x ; differentiable for $x \neq 1$; increasing for $x > 1$; decreasing for $x < 1$; concave up for $x > 1$; concave down nowhere; critical point at $x = 1$; no points of inflection; $y = 1 - x$ is an asymptote as $x \rightarrow -\infty$.
- (7) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 1$; decreasing for $x > 1$; concave up nowhere; concave down for all x ; critical point at $x = 1$; no points of inflection; no asymptotes.
- (8) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 1/2$; decreasing for $x > 1/2$; concave up nowhere; concave down for all x ; critical point at $x = 1/2$; no points of inflection; no asymptotes.
- (9) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x > 1/3$; decreasing for $x < 1/3$; concave up for all x ; concave down nowhere; -critical point at $x = 1/3$; no points of inflection; no asymptotes.
- (10) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for all x ; decreasing nowhere; concave up for $x > 0$; concave down for $x < 0$; critical point at $x = 0$; point of inflection at $x = 0$; no asymptotes.
- (11) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < -1/\sqrt{3}, x > 1/\sqrt{3}$; decreasing for $-1/\sqrt{3} < x < 1/\sqrt{3}$; concave up for $x > 0$; concave down for $x < 0$; critical points at $x = \pm 1/\sqrt{3}$; point of inflection at $x = 0$; no asymptotes.
- (12) Same as (11).
- (13) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 4/3, x > 2$; decreasing for $4/3 < x < 2$; concave up for $x > 5/3$; concave down for $x < 5/3$; critical points at $x = 2$ and $x = 4/3$; point of inflection at $x = 5/3$; no asymptotes.
- (14) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 1, x > 6$; decreasing for $1 < x < 6$; concave up for $x > 7/2$; concave down for $x < 7/2$; critical points at $x = 6$ and $x = 1$; point of inflection at $x = 7/2$; no asymptotes.
- (15) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < -5/3, x > 2$; decreasing for $-5/3 < x < 2$; concave up for $x > 1/6$; concave down for $x < 1/6$; critical points at $x = -5/3$ and $x = 2$; point of inflection at $x = 1/6$; no asymptotes.

- (16) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 0$; decreasing for $x > 0$; concave up nowhere; concave down for all x ; critical points at $x = 0$; no points of inflection; no asymptotes.
- (17) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $-1 < x < 0$ and $x > 2$; decreasing for $x < -1$ and $0 < x < 2$; concave up for x less than about $-1/2$, and for x greater than about 1.2 ; concave down for x between about $-1/2$ and 1.2 ; critical points at $x = -1$, $x = 0$ and $x = 2$; points of inflection at about $-1/2$ and about 1.2 ; no asymptotes.
- (18) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $0 < x < 1$ and $x > 3$; decreasing for $x < 0$ and $1 < x < 3$; concave up for x less than about $1/2$, and for x greater than about 2.2 ; concave down for x between about $1/2$ and 2.2 ; critical points at $x = 0$, $x = 1$ and $x = 3$; points of inflection at about $1/2$ and about 2.2 ; no asymptotes.
- (20) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x < 1.2$ and $x > 2$; decreasing for $1.2 < x < 2$; concave up for x between 0 and about 1 , and for x greater than about 1.3 ; concave down for $x < 0$ and x between about 1 and 1.3 ; critical points at $x = 0$, $x = 1.2$ and $x = 2$; points of inflection at $x = 0$ and x about 1 and about 1.3 ; no asymptotes.
- (21) Defined for all x ; continuous for all x ; differentiable for all x ; increasing for $x > 0$ and less than about 1.2 and for $x > 2$; decreasing for $x < 0$ and between about 1.2 and 2 ; concave up for x less than about $-.9$ and between about $-.5$ and $.5$ and for x greater than about 1.5 ; concave down for x between about $-.9$ and $-.5$ and between about $.5$ and 1.5 ; critical points at $x = 0$, $x = 2$ and approximately $-.9$ and 1.2 ; points of inflection at approximately $-.9$, $-.5$, $.5$ and 1.5 ; no asymptotes.