

**MATH 221: Calculus and Analytic Geometry**  
**Prof. Ram, Fall 2006**

**HOMEWORK 8: SELECTED ANSWERS**

**Problem A. Tangents and normals.**

- (1)  $10x + y - 5 = 0, \quad x - 10y + 50 = 0$
- (2)  $y = 1, \quad x = \pi/4$
- (3)  $y - 1 - (1/\sqrt{2}) = (1 - \sqrt{2})(x - \pi/4 - 1/\sqrt{2})$
- (4)  $x/a + y/b = \sqrt{2}$
- (5)  $bx \cos t + ay \sin t = ab, \quad ax \sec t - by \csc t = (a^2 - b^2)$
- (6)  $bx \sec t - ay \tan t = ab, \quad ax \cos t + by \cot t = (a^2 + b^2)$
- (7)  $y = (x - at) \tan(t/2), \quad (y - 2a) \tan(t/2) + x - at = 0$
- (8)  $8x + 3\sqrt{5}y - 36 = 0, \quad 9\sqrt{5}x - 24y + 14\sqrt{5}$
- (9)  $3y - 48\sqrt{3}x + 16\pi\sqrt{3} - 21 = 0$
- (10)  $24y - 2x + \pi - 96 = 0$
- (11)  $x + (2 + \sqrt{2})yi - 4 - 3\sqrt{2} - \pi/4 = 0$
- (14)  $(0, 0), (2a, -2a)$
- (16)  $(0, 0), (1, 2), (-1, -2)$
- (17)  $(2, 3), (-2, -3)$
- (18)  $(1, 7)$
- (19)  $x + 3y - 9 = 0$
- (20)  $2x + y - 6 = 0$

**Problem B. Optimization.**

- (1)  $(1/5, 4)$                       (2)  $(3, 9)$                       (3)  $(-4, 6)$
- (4)  $(k\pi + \pi/4, 6)$  and  $(k\pi + 3\pi/4, 4)$  where  $k$  is an integer
- (5)  $(k\pi/2 + \pi/8, 4)$  and  $(k\pi/2 + 3\pi/8, 2)$  where  $k$  is an integer

- (6)  $(1, 68), (-6, -1647), (5, -316)$       (7)  $(-2, 0), (0, -4)$   
 (8)  $(1, 0), (-1, 0), (-1/5, -3456/3125)$       (9)  $(2, 2)$   
 (10)  $(-2, 139), (3, 89)$       (11)  $(\pi/6, 3/4), (\pi/2, 1/2)$   
 (14) 49      (15)  $(1, 3)$   
 (16)  $(\pi/3, -\pi/3 + \sqrt{3}), (5\pi/3, -(5\pi/3 + \sqrt{3}))$   
 (17) 6, 9      (18)  $\frac{ap}{p+q}, \frac{aq}{p+q}$       (20)  $(1/p)^{1/(p-1)}$   
 (21) length  $4\sqrt{6}$  cm, width  $4\sqrt{6}$  cm, perimeter  $16\sqrt{6}$  cm  
 (25)  $r = (500/\pi)^{1/3}$  cm,  $h = \frac{1000}{\pi^{1/3}500^{1/3}}$  cm  
 (26)  $p^3/6\sqrt{3}$  cubic units      (28) 4, 4      (29)  $75 \times 75$  yards

**Problem C. Related rates.**

- (1)  $\frac{dV}{dr} = 4\pi r^2$       (2)  $\frac{dV}{dr} = 2\pi r h$   
 (3)  $\frac{dS}{dr} = \frac{\pi(h^2 + 2r^2)}{\sqrt{h^2 + r^2}}$       (4)  $\frac{dP}{dt} = 0.8$  cm/s  
 (5)  $\frac{dr}{dt} \approx 0.32$  cm/s      (6)  $\frac{dV}{dt} = 6$  cm<sup>3</sup>/s  
 (7)  $\frac{dh}{dt} = 0.5$  cm/min      (8) 6.78 m/s  
 (9) (a)  $25/3$  ft/s    (b)  $10/3$  ft/s      (10)  $80\pi/3$  km/min  
 (11)  $\sqrt{65}/8$  m/sec      (12)  $6/5\pi$  ft/min  
 (13)  $32/27\pi$  cm/s      (14)  $\approx 2.89 \times 10^5$  cm<sup>3</sup>/min  
 (15)  $16/49\pi$  cm/sec      (16)  $\sqrt{2}/5$  rad/sec  
 (17)  $5/6$  m/s      (18) 0.032 rad/s  
 (19)  $\pi/3$  radians      (20) (a) 360 ft/s    (b) 0.096 rad/s