

MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 5
DUE October 9, 2006

Problem A. Evaluating limits when $x \rightarrow 0$.

- (1) Evaluate $\lim_{x \rightarrow 0} (x^2 - 2)^2 + 6$.
- (2) Evaluate $\lim_{x \rightarrow 0} \frac{5x}{x}$.
- (3) Evaluate $\lim_{x \rightarrow 0} \frac{17x}{2x}$.
- (4) Evaluate $\lim_{x \rightarrow 0} \frac{-317x}{422x}$.
- (5) Evaluate $\lim_{x \rightarrow 0} \frac{-317x - 3}{422x + 5}$.
- (6) Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$.
- (7) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$.
- (8) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$.
- (9) Evaluate $\lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right)$.
- (10) Evaluate $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$.
- (11) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.
- (12) Evaluate $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - 1}$.
- (13) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{x^2}$.

(14) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \sqrt{ax + b}$.

(15) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = (mx + c)^n$.

Problem B. Evaluating limits when $x \rightarrow a$.

(1) Evaluate $\lim_{x \rightarrow 1} (6x^2 - 4x + 3)$.

(2) Evaluate $\lim_{x \rightarrow 7} \frac{x^2 - 49}{x - 7}$.

(3) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2}$.

(4) Evaluate $\lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$.

(5) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$.

(6) Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$.

(7) Evaluate $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$.

(8) Evaluate $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$.

(9) Evaluate $\lim_{x \rightarrow 5} \frac{x^5 - 3125}{x - 5}$.

(10) Evaluate $\lim_{x \rightarrow a} \frac{x^{12} - a^{12}}{x - a}$.

(11) Evaluate $\lim_{x \rightarrow a} \frac{x^{5/2} - a^{5/2}}{x - a}$.

(12) Evaluate $\lim_{x \rightarrow a} \frac{(x + 2)^{5/3} - (a + 2)^{5/3}}{x - a}$.

(13) Evaluate $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$.

(14) Evaluate $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8}$.

(15) Evaluate $\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}$.

(16) Evaluate $\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$.

(17) Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$.

(18) Evaluate $\lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$.

(19) Evaluate $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$.

Problem C. Evaluating limits as $x \rightarrow \infty$.

(1) Evaluate $\lim_{x \rightarrow \infty} \frac{x+2}{x-2}$.

(2) Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 5}{5x^2 + 3x + 1}$.

(3) Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 - 7x + 11}{3x^2 + 10}$.

(4) Evaluate $\lim_{x \rightarrow \infty} \frac{2x^3 - 5x + 7}{7x^3 + 2x^2 - 6}$.

(5) Evaluate $\lim_{x \rightarrow \infty} \frac{(3x-1)(4x-5)}{(x+6)(x-3)}$.

(6) Evaluate $\lim_{n \rightarrow \infty} \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^n}$.

(7) Evaluate $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1} - 1}$.

(8) Evaluate $\lim_{x \rightarrow -\infty} 2^x$.

(9) Evaluate $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$.

- (10) Evaluate $\lim_{t \rightarrow \infty} \frac{t+1}{t^2+1}$.
- (11) Evaluate $\lim_{n \rightarrow \infty} \sqrt{n^2+1} - n$.
- (12) Evaluate $\lim_{n \rightarrow \infty} \sqrt{n^2+n} - n$.

Problem D. Limits with exponential and log functions.

- (1) Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$.
- (2) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$.
- (3) Evaluate $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$.
- (4) Evaluate $\lim_{x \rightarrow 0} (1+x)^{1/x}$.
- (5) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$.
- (6) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{x^2}$.
- (7) Evaluate $\lim_{x \rightarrow -\infty} 2^x$.
- (8) Explain why $\lim_{x \rightarrow -1} \ln x$ does not exist.
- (9) Explain why $\lim_{x \rightarrow 0} 2^{1/x}$ does not exist.
- (10) Explain why $\lim_{x \rightarrow 1} 2^{1/(x-1)}$ does not exist.
- (11) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = e^{\sqrt{x}}$.
- (12) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \ln(ax+b)$.
- (13) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = x^x$.

Problem E. Limits with trigonometric functions.

(1) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{4x}$.

(2) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$.

(3) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x}{x}$.

(4) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$.

(5) Evaluate $\lim_{x \rightarrow 0} \frac{\tan ax}{\tan bx}$.

(6) Evaluate $\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x}$.

(7) Evaluate $\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx}$.

(8) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos 6\theta}{\theta}$.

(9) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x}$.

(10) Evaluate $\lim_{x \rightarrow 0} \frac{\cos^2 x}{1 - \sin x}$.

(11) Evaluate $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$.

(12) Evaluate $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$.

(13) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}$.

(14) Evaluate $\lim_{x \rightarrow 0} \frac{\tan 3x - 2x}{3x - \sin^2 x}$.

(15) Evaluate $\lim_{x \rightarrow 0} \frac{x^2 - \tan 2x}{\tan x}$.

(16) Evaluate $\lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{x - \pi/4}$.

(17) Evaluate $\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x}$.

(18) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x + \tan^2 x}{x \sin x}$.

(19) If $\lim_{x \rightarrow 0} kx \csc x = \lim_{x \rightarrow 0} x \csc kx$, explain why $k = \pm 1$.

(20) Evaluate $\lim_{h \rightarrow 0} \frac{\sin(a+h) - \sin a}{h}$.

(21) Evaluate $\lim_{h \rightarrow \infty} \frac{\cos(\pi/h)}{h-2}$.

Problem F. Calculating $\lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}$.

(1) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \sin 2x$.

(2) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \cos 2x$.

(3) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \sin x^2$.

(4) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \cos x^2$.

(5) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \sqrt{\sin x}$.

(6) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = x \sin x$.

(7) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = e^{\sin x}$.

(8) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = e^{\cos x}$.

(9) Calculate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ when $f(x) = \frac{\sin x}{x}$.

Problem G. Limits with inverse trig functions.

(1) Evaluate $\lim_{x \rightarrow 1} \frac{1 - x}{(\cos^{-1} x)^2}$.

(2) Evaluate $\lim_{x \rightarrow 1/\sqrt{2}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$.

(3) Evaluate $\lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1 - x^2})}{(\sin^{-1} x)^3 \sqrt{1 - x^2}}$.

(4) Evaluate $\lim_{x \rightarrow 1} \frac{1 - x}{\pi - 2 \sin^{-1} x}$.

(5) Evaluate $\lim_{x \rightarrow 0} \frac{\tan^{-1} 2x}{\sin 3x}$.