MATH 221 Lecture 17, October 16, 2000 ①

Graphing Examples

Example Graph $f(x) = 3x^2 - 2x - 1$

Notes

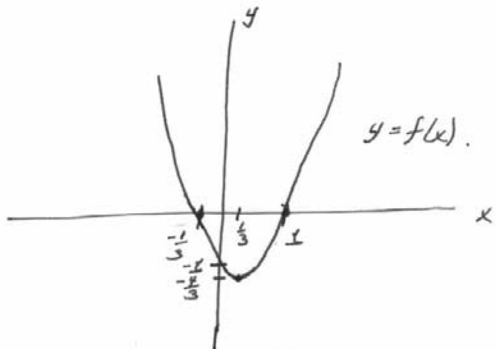
(a) The x^2 indicates this is a parabola.

(b) Since the coefficient of x^2 is positive this is a concave up parabola.

(c) $3x^2 - 2x - 1 = (x-1)(3x+1)$.

We knew $x-1$ should be a factor since when you plug in 1, $3\cdot 1^2 - 2\cdot 1 - 1 = 0$.

(d) $f(x) = 0$ if $x = 1$ or if $x = -\frac{1}{3}$



(e) The minimum will be where $\left.\frac{df}{dx}\right|_{x=a}$ is 0.

$$\left.\frac{df}{dx}\right|_{x=a} = \left.6x - 2\right|_{x=a} = 6a - 2. \text{ This's 0 when } a = \frac{1}{3}$$

$$f\left(\tfrac{1}{3}\right) = 3\left(\tfrac{1}{3}\right)^2 - 2\left(\tfrac{1}{3}\right) - 1 = \frac{1}{3} - \frac{2}{3} - 1 = -\frac{4}{3}$$

Example Graph $f(x) = 2x^3 - 21x^2 + 36x - 20$. ②

Notes:

(a) If $x \to \infty$ then $f(x) \to \infty$

(b) If $x \to -\infty$ then $f(x) \to -\infty$.

(c) $\frac{df}{dx} = 6x^2 - 42x + 36 = 6(x^2 - 7x + 6) = 6(x-6)(x-1)$

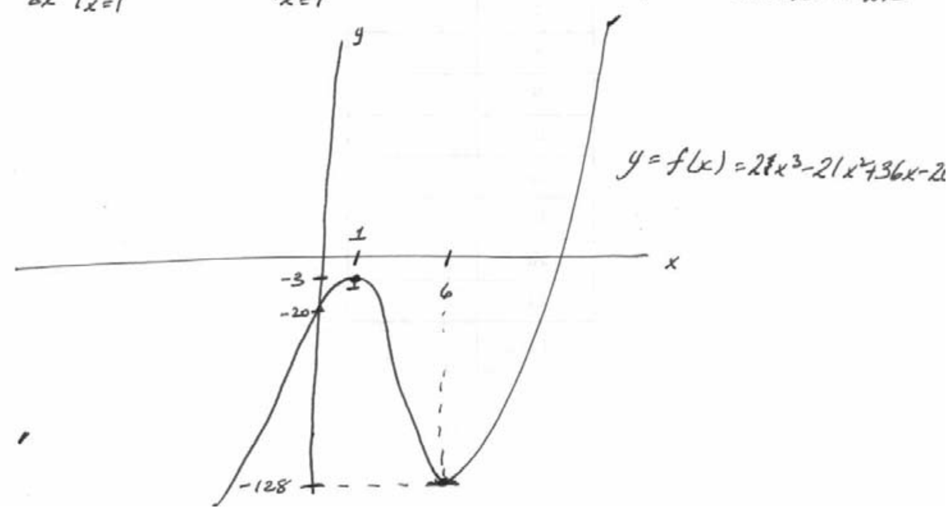
So $\frac{df}{dx}$ is 0 when $x = 6$ and when $x = 1$.

$$f(6) = 2\cdot 6^3 - 21\cdot 6^2 + 36\cdot 6 - 20 = 6^2(12 - 21 + 6) - 20$$
$$= 6^2(-3) - 20 = -128$$

$$f(1) = 2 - 21 + 36 - 20 = 38 - 41 = -3.$$

(d) $\left.\frac{d^2f}{dx^2}\right|_{x=6} = \left.12x - 42\right|_{x=6} = 72 - 42 = 30 > 0.$ concave up

$\left.\frac{d^2f}{dx^2}\right|_{x=1} = \left.12x - 42\right|_{x=1} = 12 - 42 = -30 < 0.$ concave down

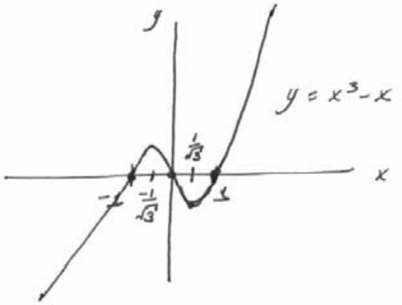


<u>Example</u> Graph $f(x) = x^3 - x + 1$.

<u>Notes</u>

(a) This is $x^3 - x$ shifted up by 1.

(b) $x^3 - x = x(x^2-1) = x(x+1)(x-1)$

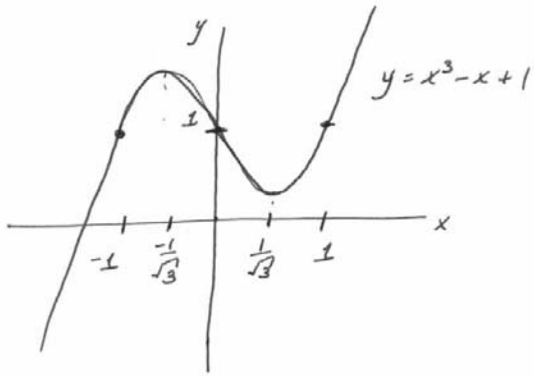


$\frac{d(x^3-x)}{dx} = 3x^2 - 1$

So $\left.\frac{d(x^3-x)}{dx}\right|_{x=a}$ is 0 when

$a = \pm\frac{1}{\sqrt{3}}$

So



<u>Example</u> Graph $x - x^2 - 27$.

<u>Notes</u>

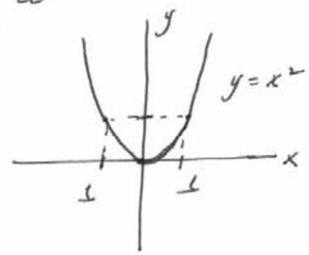
(a) The $-x^2$ indicates to us that this is a concave down parabola.

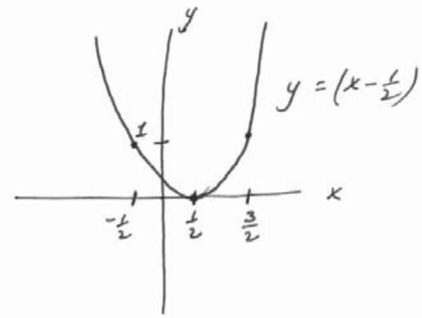
(b) $x - x^2 - 27 = -(x^2 - x + 27)$

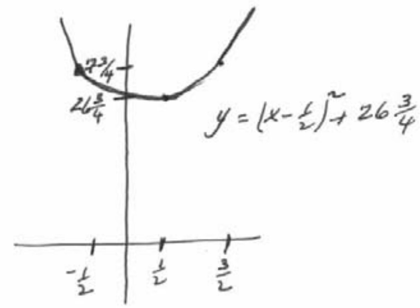
$= -(x^2 - x + \frac{1}{4} - \frac{1}{4} + 27)$

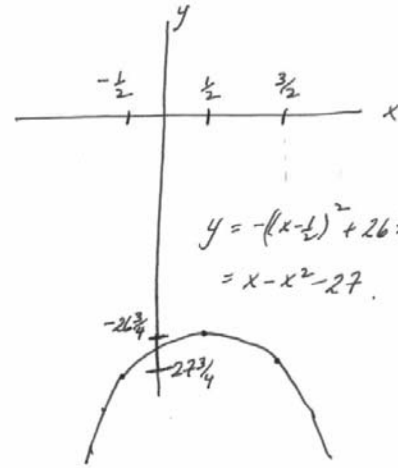
$= -\left((x - \frac{1}{2})^2 + 26\frac{3}{4}\right)$

So









$y = -((x - \frac{1}{2})^2 + 26\frac{3}{4})$

$= x - x^2 - 27$.

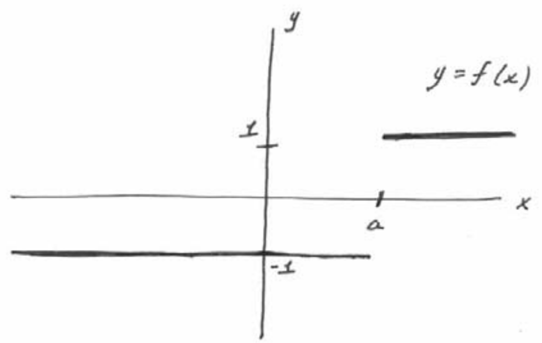
<u>Example</u> For which values of x is

$$f(x) = \begin{cases} \frac{|x-a|}{x-a}, & \text{if } x \neq a \\ 1, & \text{if } x = a \end{cases}$$ continuous?

<u>Notes:</u>

$$f(x) = \begin{cases} \frac{|x-a|}{x-a}, & \text{if } x \neq a \\ 1, & \text{if } x = a \end{cases} = \begin{cases} \frac{x-a}{x-a}, & \text{if } x \neq a \text{ and } x-a > 0 \\ \frac{-(x-a)}{x-a}, & \text{if } x \neq a \text{ and } x-a < 0 \\ 1, & \text{if } x = a \end{cases}$$

$$= \begin{cases} 1, & \text{if } x \neq a \text{ and } x > a \\ -1, & \text{if } x \neq a \text{ and } x < a \\ 1, & \text{if } x = a \end{cases}$$



$f(x)$ has a jump at $x = a$.

So $f(x)$ is not continuous at $x = a$.