(1)

MATH 221 Lecture 22 October 30, 2000.

Integrals

$$f \to \boxed{\frac{df}{dx}} \to \frac{df}{dx} \qquad \frac{df}{dx} \to \boxed{\int \; dx} \to f$$

The integral with respect to x undoes the derivative with respect to x.

$$\int 5x^4\,dx = x^5, \qquad \text{since } \frac{dx^5}{dx} = 5x^4$$

ALSO

$$\int 5x^4\,dx = x^5+2, \qquad \text{since } \frac{d(x^5+2)}{dx} = 5x^4.$$

This is like $\sqrt{9} = 3$ and $\sqrt{9} = -3$.

We usually write

$$\int 5x^4\,dx = x^5 + c, \quad \text{where } c \text{ is a constant,}$$

so that we can write all possible answers to $\int 5x^4\,dx$ at once.

$$\int x^2\,dx = \frac{x^3}{3} + c, \qquad \text{since } \frac{d\left(\frac{x^3}{3}\right)}{dx} = \frac{1}{3}3x^2 = x^2.$$

(2)

$$\int x^3\,dx = \frac{x^4}{4} + c, \qquad \text{since } \frac{d\left(\frac{x^4}{4}\right)}{dx} = \frac{1}{4}4x^3 = x^3$$

$$\int x^4\,dx = \frac{x^5}{5} + c, \qquad \text{since } \frac{d\left(\frac{x^5}{5}\right)}{dx} = \frac{1}{5}5x^4 = x^4.$$

$$\int x^5\,dx = \frac{x^6}{6} + c, \qquad \text{since } \frac{d\left(\frac{x^6}{6}\right)}{dx} = \frac{1}{6}6x^5 = x^5.$$

$$\int x^{642}\,dx = \frac{x^{643}}{643} + c, \qquad \text{since } \frac{d\left(\frac{x^{643}}{643}\right)}{dx} = \frac{1}{643}x^{642}\cdot 643$$

$$= x^{642}.$$

$$\int x^{-2}\,dx = \frac{x^{-1}}{-1} + c, \qquad \text{since } \frac{d\left(\frac{x^{-1}}{-1}\right)}{dx} = \frac{1}{-1}(-1)x^{-2} = x^{-2}.$$

$$\int x^{-3}\,dx = \frac{x^{-2}}{-2} + c, \qquad \text{since } \frac{d\left(\frac{x^{-2}}{-2}\right)}{dx} = \frac{1}{(-2)}(-2)x^{-3} = x^{-3}.$$

$$\int x^{-642}\,dx = \frac{x^{-641}}{-641} + c, \qquad \text{since } \frac{d\left(\frac{x^{-641}}{-641}\right)}{dx} = \frac{1}{-641}(-641)x^{-642}$$

$$= x^{-642}.$$

$$\int dx = \int 1\,dx = \int x^0\,dx = \frac{x^1}{1} + c, \qquad \text{since } \frac{dx}{dx} = 1.$$

$\int x^{-1}dx = \int \frac{1}{x}\,dx = \ln x + c$, since $\frac{d \ln x}{dx} = \frac{1}{x}$.

Note: $\int x^{-1}dx$ is NOT $\frac{x^0}{0} + c$; $\frac{x^0}{0}$ DOESN'T MAKE SENSE.

Example $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}dx = \frac{x^{3/2}}{3/2} + c = \frac{2}{3}x^{3/2} + c$,

since $\frac{d\,\frac{2}{3}x^{3/2}}{dx} = \frac{2}{3}\,\frac{3}{2}\,x^{\frac{1}{2}} = x^{\frac{1}{2}}$.

Example $\int \sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}dx = \frac{x^{4/3}}{4/3} + c = \frac{3}{4}x^{4/3} + c$,

since $\frac{d\,\frac{3}{4}x^{4/3}}{dx} = \frac{3}{4}\cdot\frac{4}{3}x^{4/3-1} = x^{\frac{1}{3}}$.

Example $\int x^{\frac{362}{431}}\,dx = \frac{x^{\frac{362}{431}+1}}{\frac{362}{431}+1} + c = \frac{x^{\frac{793}{431}}}{\frac{793}{431}} + c$

$= \frac{431}{793}\,x^{793/431} + c$, since $\frac{d\,\frac{431}{793}x^{\frac{793}{431}}}{dx} = \frac{431}{793}\,\frac{793}{431}\,x^{\frac{793}{431}-1}$

$= x^{362/431}$.

$\int e^x dx = e^x + c$, since $\frac{d e^x}{dx} = e^x$.

$\int e^{10x}dx = \frac{e^{10x}}{10} + c$, since $\frac{d\left(\frac{e^{10x}}{10}\right)}{dx} = \frac{1}{10}e^{10x}\cdot 10 = e^{10x}$.

$\int e^{-31x}dx = \frac{e^{-31x}}{-31} + c$, since $\frac{d\left(\frac{e^{-31x}}{-31}\right)}{dx} = \frac{1}{-31}e^{-31x}(-31) = e^{-31x}$.

Example $\int 2^x dx = \int (e^{\ln 2})^x dx = \int e^{x\ln 2}dx = \frac{e^{x\ln 2}}{\ln 2} + c$

$= \frac{2^x}{\ln 2} + c$, since $\frac{d\,\frac{2^x}{\ln 2}}{dx} = \frac{d\,\frac{e^{x\ln 2}}{\ln 2}}{dx} = \frac{1}{\ln 2}e^{x\ln 2}\cdot\ln 2 = e^{x\ln 2} = 2^x$.

Example $\int 38^x dx = \int (e^{\ln 38})^x dx = \int e^{x\ln 38}dx = \frac{e^{x\ln 38}}{\ln 38} + c$,

since $\frac{d\,\frac{e^{x\ln 38}}{\ln 38}}{dx} = \frac{1}{\ln 38}e^{x\ln 38}\cdot\ln 38 = e^{x\ln 38} = 38^x$.

$\int \sin x\,dx = -\cos x + c$, since $\frac{d(-\cos x)}{dx} = -(-\sin x)$
$= \sin x$.

$\int \cos x\,dx = \sin x + c$, since $\frac{d \sin x}{dx} = \cos x$.

$\int \sec^2 x\,dx = \tan x + c$, since $\frac{d \tan x}{dx} = \sec^2 x$.

$\int \csc^2 x\, dx = -\cot x + c$, since $\dfrac{d(-\cot x)}{dx} = -(-\csc^2 x)$

$= \csc^2 x.$

$\int \tan x \sec x\, dx = \sec x + c$, since $\dfrac{d\sec x}{dx} = \tan x \sec x.$

$\int \cot x \csc x\, dx = -\csc x + c$, since $\dfrac{d(-\csc x)}{dx} = -(-\cot x \csc x)$

$= \cot x \csc x.$

<u>Example</u> $\displaystyle\int \frac{\cos x}{\sin^2 x}\, dx = \int \frac{\cos x}{\sin x}\,\frac{1}{\sin x}\, dx$

$\displaystyle = \int \cot x \csc x\, dx = -\csc x + c.$

<u>Example</u> $\displaystyle\int \frac{1}{1+\cos x}\, dx = \int \frac{1}{(1+\cos x)}\,\frac{(1-\cos x)}{(1-\cos x)}\, dx$

$\displaystyle = \int \frac{1-\cos x}{1-\cos^2 x}\, dx = \int \frac{1-\cos x}{\sin^2 x}\, dx = \int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}\right) dx$

$\displaystyle = \int (\csc^2 x - \cot x \csc x)\, dx = -\cot x + \csc x + c.$