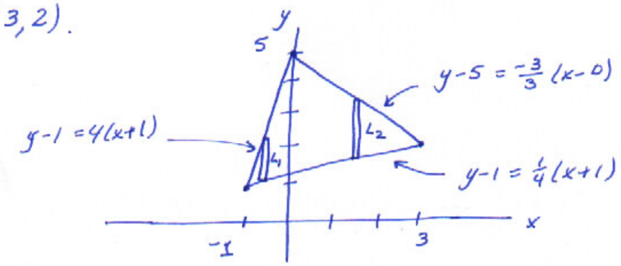
MATH 221 Lecture 27, November 10, 2000 ①

Example Use integration to find the area of the triangle with vertices $(-1,1)$, $(0,5)$ and $(3,2)$.



Type 1 slice: L_1, dx — Type 2 slice: L_2, dx

Area of type 1 slice: $L_1\,dx$ — Area of type 2 slice: $L_2\,dx$

Add slices from $x=-1$ to $x=0$ — Add slices from $x=0$ to $x=3$

$$\int_{x=-1}^{x=0} L_1\,dx + \int_{x=0}^{x=3} L_2\,dx = \int_{x=-1}^{x=0} (y_{top1}-y_{bottom})\,dx + \int_{x=0}^{x=3} (y_{top2}-y_{bottom})\,dx$$

$$= \int_{x=-1}^{x=0} \left((4(x+1)+1)-\left(\tfrac{1}{4}(x+1)+1\right)\right)dx + \int_{x=0}^{x=3} \left((-x+5)-\left(\tfrac{1}{4}(x+1)+1\right)\right)dx$$

$$= \int_{x=-1}^{x=0} \left(4x+5-\tfrac{1}{4}x-\tfrac{1}{4}-1\right)dx + \int_{x=0}^{x=3} \left(-x+5-\tfrac{1}{4}x-\tfrac{1}{4}-1\right)dx$$

$$= \int_{x=-1}^{x=0} \left(\tfrac{15}{4}x+\tfrac{15}{4}\right)dx + \int_{x=0}^{x=3} \left(\tfrac{-5}{4}x+\tfrac{15}{4}\right)dx$$

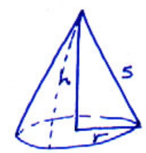
②

$$= \left(\frac{15}{4}\frac{x^2}{2}+\frac{15}{4}x\right)\Bigg|_{x=-1}^{x=0} + \left(\frac{-5}{4}\frac{x^2}{2}+\frac{15}{4}x\right)\Bigg|_{x=0}^{x=3}$$

$$= \left(\frac{15}{4}\cdot 0+\frac{15}{4}\cdot 0\right)-\left(\frac{15}{4}\frac{(-1)^2}{2}+\frac{15}{4}(-1)\right)+\left(\frac{-5}{4}\cdot\frac{3^2}{2}+\frac{15}{4}\cdot 3\right)-(0+0)$$

$$= 0+0-\frac{15}{8}+\frac{15}{4}-\frac{45}{8}+\frac{45}{4} = \frac{15}{8}+\frac{45}{8} = \frac{60}{8} = 7\tfrac{1}{2}.$$

Example Find the curved surface area of a cone of radius r and height h (a right circular cone).



Cut the cone open and lay it out to get

The region C is a portion of a circle of radius s, where s is the slant height of the cone. The area of C is $\frac{1}{2}\theta s^2$.

The arc length along the border of C is θs.

This arc length is also the length around the cirle at the base of the cone, which is $2\pi r$.

So $\theta s = 2\pi r$.

So (3)

curved surface area $= \frac{1}{2}\theta s^2$

$$= \frac{1}{2}(\theta s)s$$

$$= \frac{1}{2}(2\pi r)s$$

$$= \pi r s$$

$$= \pi r\sqrt{h^2+r^2}$$