MATH 222 Lecture 3 September 11, 2000

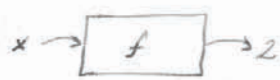
A function takes in a number, chews on it and spits out another number.

A constant function always spits out the same number, no matter what the input is.

Example $f(x)=2$

If $x=3\pi+7$ then $f(x)=2$

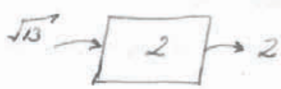
If $x=3+3i$ then $f(x)=2$



If $x=1$ then $f(x)=2$

If $x=\sqrt{7}$ then $f(x)=2$

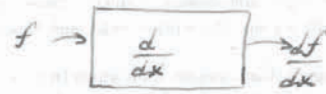
We call this function 2.



So 2 sometimes means the number 2 and sometimes means the function 2.

Derivatives

A derivative takes in a function, chews on it and spits out another function



The derivative $\frac{d}{dx}$ spits out a function according to the rules:

(1) $\frac{dx}{dx}=1$,

(2) $\frac{d(cf)}{dx}=c\frac{df}{dx}$, if c does not change when x changes

(3) $\frac{d(f+g)}{dx}=\frac{df}{dx}+\frac{dg}{dx}$

(4) $\frac{d(fg)}{dx}=f\frac{dg}{dx}+\frac{df}{dx}g$

Example Find $\frac{dy}{dx}$ if $y=5x$.

$$\frac{dy}{dx}=\frac{d(5x)}{dx}=5\frac{dx}{dx}=5\cdot 1=5.$$

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Example Find $\frac{dy}{dx}$ if $y = \pi x$

$$\frac{dy}{dx} = \frac{d(\pi x)}{dx} = \pi \frac{dx}{dx} = \pi \cdot 1 = \pi.$$

Example Find $\frac{dy}{dx}$ if $y = 1$.

$$\frac{dy}{dx} = \frac{d1}{dx} = \frac{d(1 \cdot 1)}{dx} = 1\frac{d1}{dx} + \frac{d1}{dx} \cdot 1 = \frac{d1}{dx} + \frac{d1}{dx}.$$

Subtract $\frac{d1}{dx}$ from both sides

So $\frac{d1}{dx} = 0$

Example Find $\frac{dy}{dx}$ if $y = 5$.

$$\frac{dy}{dx} = \frac{d5}{dx} = \frac{d(5 \cdot 1)}{dx} = 5\frac{d1}{dx} = 5 \cdot 0 = 0$$

Example Find $\frac{dy}{dx}$ if $y = 6342$

$$\frac{d\,6342}{dx} = \frac{d(6342 \cdot 1)}{dx} = 6342\frac{d1}{dx} = 6342 \cdot 0 = 0.$$

Example Find $\frac{dc}{dx}$ if c is a constant

$$\frac{dc}{dx} = \frac{d(c \cdot 1)}{dx} = c\frac{d1}{dx} = c \cdot 0 = 0.$$

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Example Find $\frac{dy}{dx}$ if $y = 3x + 12$.

$$\frac{dy}{dx} = \frac{d(3x+12)}{dx} = \frac{d(3x)}{dx} + \frac{d12}{dx} = 3\frac{dx}{dx} + 0 = 3 \cdot 1 + 0 = 3.$$

Example Find $\frac{dy}{dx}$ if $y = x^2$.

$$\frac{dy}{dx} = \frac{dx^2}{dx} = \frac{d\,x \cdot x}{dx} = x\frac{dx}{dx} + \frac{dx}{dx} \cdot x = x \cdot 1 + 1 \cdot x = 2x.$$

Example Find $\frac{dx^3}{dx}$.

$$\frac{dx^3}{dx} = \frac{d(x^2 \cdot x)}{dx} = x^2\frac{dx}{dx} + \frac{dx^2}{dx} \cdot x = x^2 \cdot 1 + 2x \cdot x = 3x^2.$$

Example Find $\frac{dx^4}{dx}$

$$\frac{dx^4}{dx} = \frac{d(x^3 \cdot x)}{dx} = x^3\frac{dx}{dx} + \frac{dx^3}{dx} \cdot x = x^3 \cdot 1 + 3x^2 \cdot x = 4x^3$$

Example Find $\frac{dx^{6342}}{dx}$

$$\frac{dx^{6342}}{dx} = \frac{d(x^{6341} \cdot x)}{dx} = x^{6341}\frac{dx}{dx} + \frac{dx^{6341}}{dx} \cdot x$$

$$= x^{6341} \cdot 1 + 6341x^{6340} \cdot x$$

$$= x^{6341} + 6341x^{6341} = 6342x^{6341}.$$

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Example Find $\frac{dx^n}{dx}$ if $n = 1, 2, 3, \ldots$

$$\frac{dx^n}{dx} = \frac{d(x^{n-1} \cdot x)}{dx} = x^{n-1}\frac{dx}{dx} + \frac{dx^{n-1}}{dx} \cdot x$$

$$= x^{n-1} \cdot 1 + (n-1)x^{n-2} \cdot x$$

(since we just found ... $\frac{dx^{n-1}}{dx} = (n-1)x^{n-2}$)

$$= x^{n-1} + (n-1)x^{n-1}$$

So

$$\frac{dx^n}{dx} = nx^{n-1}$$

Example Find $\frac{dx^n}{dx}$ if $n = 0$

$$\frac{dx^n}{dx} = \frac{dx^0}{dx} = \frac{d1}{dx} = 0 = nx^{n-1}$$

Example Find $\frac{dx^{-6342}}{dx}$

$$\frac{d(x^{-6342} \cdot x^{6342})}{dx} = \frac{dx^0}{dx} = \frac{d1}{dx} = 0$$

On the other hand,

$$\frac{d(x^{-6342} \cdot x^{6342})}{dx} = x^{-6342}\frac{dx^{6342}}{dx} + \frac{dx^{-6342}}{dx} \cdot x^{6342}$$

$$= x^{-6342}\, 6342 x^{6341} + \frac{dx^{-6342}}{dx} \cdot x^{6342}.$$

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So

$$0 = 6342x^{-1} + x^{6342}\frac{dx^{-6342}}{dx}$$

So

$$\frac{dx^{-6342}}{dx} = -6342x^{-1}x^{-6342}$$

$$= (-6342)x^{-6343}$$

Example Find $\frac{dx^{-n}}{dx}$ if $n = 1, 2, 3, \ldots$

$$\frac{dx^n x^{-n}}{dx} = x^n\frac{dx^{-n}}{dx} + \frac{dx^n}{dx}x^{-n} = x^n\frac{dx^{-n}}{dx} + nx^{n-1}x^{-n}$$

$$= x^n\frac{dx^{-n}}{dx} + nx^{-1}$$

On the other hand

$$\frac{d(x^n x^{-n})}{dx} = \frac{dx^0}{dx} = \frac{d1}{dx} = 0.$$

So

$$x^n\frac{dx^{-n}}{dx} + nx^{-1} = 0.$$

Solve for $\frac{dx^{-n}}{dx}$. Then

$$\frac{dx^{-n}}{dx} = -nx^{-1}x^{-n} = -nx^{-n-1}$$

Example Let $y = 3x^3+5x^2+2x+7$. Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d(3x^3+5x^2+2x+7)}{dx} = \frac{d(3x^3)}{dx} + \frac{d(5x^2+2x+7)}{dx}$$

$$= \frac{d(3x^3)}{dx} + \frac{d(5x^2)}{dx} + \frac{d(2x)}{dx} + \frac{d7}{dx}$$

$$= 3\frac{dx^3}{dx} + 5\frac{dx^2}{dx} + 2\frac{dx}{dx} + 7\frac{d1}{dx}$$

$$= 3\cdot 3x^2 + 5\cdot 2x + 2\cdot 1 + 7\cdot 0 = 9x^2+10x+2$$

Example If $y = -7x^{-13} + 5x^{-7} + (6+2i)x^{38}$ Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{d(-7x^{-13}+5x^{-7}+(6+2i)x^{38})}{dx}$$

$$= \frac{d(-7x^{-13})}{dx} + \frac{d(5x^{-7})}{dx} + \frac{d((6+2i)x^{38})}{dx}$$

$$= -7\frac{dx^{-13}}{dx} + 5\frac{dx^{-7}}{dx} + (6+2i)\frac{dx^{38}}{dx}$$

$$= (-7)(-13)x^{-13-1} + 5(-7)x^{-7-1} + (6+2i)38x^{38-1}$$

$$= 91x^{-14} - 35x^{-8} + (228+76i)x^{37}.$$