MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

Lectures 4 and 5: MIDTERM EXAM 1, Sample
October 9, 2006

This is a 50 minute exam. No books, notes or calculators are allowed. There are 10 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: _____

TA and Section: _____

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

Problem 1. What is $\ln x$?

Problem 2. Evaluate $\displaystyle\lim_{h \to 0} \frac{\sin(a + h) - \sin a}{h}$.

Problem 3. Explain why $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$.

$\dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$.

Problem 4. If $y = e^{-x}\cos x$ prove that $\dfrac{d^4y}{dx^4} + 4y = 0$.

Problem 5. Verify the identity $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$.

Problem 6. Evaluate $\displaystyle\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$.

$$\frac{\sqrt{2+x} - \sqrt{2}}{x}.$$

Problem 7. Verify the identity $\cos 3x = \cos^3 x - 3\cos x \sin^2 x$.

Problem 8. Verify the identity $\dfrac{\sin\alpha + \sin 3\alpha}{\cos\alpha + \cos 3\alpha} = \tan 2\alpha.$

Problem 9. Explain why $1 + \dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \dfrac{1}{3^4} + \cdots = \dfrac{3}{2}$.

Problem 10. Find $\dfrac{dy}{dx}$ when $y = \ln \dfrac{\sin^m x}{\cos^n x}$.