MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

Lectures 4 and 5: MIDTERM EXAM 2, Sample
October 30, 2006

This is a 50 minute exam. No books, notes or calculators are allowed. There are 10 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: ______________________________

TA and Section: ______________________________

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
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9.	
10.	
Total	

Problem 1. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

Problem 2. Graph $f(x) = ax + b$, where a and b are constants. Also determine

(a) where $f(x)$ is defined,
(b) where $f(x)$ is continuous,
(c) where $f(x)$ is differentiable,
(d) where $f(x)$ is increasing and where it is decreasing,
(e) where $f(x)$ is concave up and where it is concave down,
(f) what the critical points of $f(x)$ are,
(g) where the points of inflection are, and
(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 3. Graph $y = f(x)$ when $x = 2y^2 - 1$. Also determine

(a) where $f(x)$ is defined,
(b) where $f(x)$ is continuous,
(c) where $f(x)$ is differentiable,
(d) where $f(x)$ is increasing and where it is decreasing,
(e) where $f(x)$ is concave up and where it is concave down,
(f) what the critical points of $f(x)$ are,
(g) where the points of inflection are, and
(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 4. The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

Problem 5. Divide 15 into two parts such that the square of one times the cube of the other is maximum.

Problem 6. Graph $f(x) = x^{-3}$.

Problem 7. Graph $f(x) = x\sqrt{1-x^2}$. Also determine

(a) where $f(x)$ is defined,

(b) where $f(x)$ is continuous,

(c) where $f(x)$ is differentiable,

(d) where $f(x)$ is increasing and where it is decreasing,

(e) where $f(x)$ is concave up and where it is concave down,

(f) what the critical points of $f(x)$ are,

(g) where the points of inflection are, and

(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 8. Verify Rolle's theorem for the function $f(x) = (x-2)^2(x-3)^6$ on the interval $[2, 3]$.

Problem 9. For which values of x is the function $f(x) = \lfloor x \rfloor$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Problem 10. Verify the mean value theorem for the function $f(x) = \ell x^2 + mx + n$ in the interval $[a, b]$, where ℓ, m and n are constants.