University of Wisconsin-Madison
Mathematics Department

Math 541
Modern Algebra
A first course in Abstract Algebra
Lecturer: Arun Ram

Fall 2007

Homework 10: Due November 15, 2007

To grade: 4, 9, 12, 16.

1. Define G-set, stabilizer and orbit.
2. Let S be a G-set. Show that the orbits partition S.
3. Let S be a G-set and let $s \in S$. Show that the stabilizer of s is a subgroup of G.
4. Let S be a G-set and let $s \in S$. Show that there exists a bijection between G/G_s and Gs.
5. Let S be a G-set. Let $s \in S$ and $g \in G$. Show that $G_{gs} = gG_s g^{-1}$.
6. Let G be a group. The group G acts on itself by left multiplication. Compute the stabilizer and orbit of each element.
7. Define conjugacy class and centralizer and explain the relationship between these and the action of G on itself by conjugation.
8. Let G be a group and let H be a subgroup of G. The group G acts on G/H by left multiplication. Compute the stabilizer and orbit of each coset.
9. Define center and conjugacy class and prove the class equation.
10. The symmetric group S_4 acts on $S = \{1, 2, 3, 4\}$ by permutations. Compute the stablizer and the orbit of each element.
11. The dihedral group D_5 acts on the vertices of a pentagon. Compute the stabilizer and the orbit of each vertex.

12. The dihedral group D_5 acts on the edges of a pentagon. Compute the stabilizer and the orbit of each edge.

13. The cyclic group C_5 acts on the vertices of a pentagon. Compute the stabilizer and the orbit of each vertex.

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15. The symmetric group S_4 acts on the vertices of a tetrahedron. Compute the stabilizer and the orbit of each vertex.

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17. The symmetric group S_4 acts on the faces of a tetrahedron. Compute the stabilizer and the orbit of each face.

18. Describe how the group $(\mathbb{Z}/2\mathbb{Z})\times(\mathbb{Z}/2\mathbb{Z})\times(\mathbb{Z}/2\mathbb{Z})$ acts on the vertices of a cube. Compute the stabilizer and orbit of each vertex.

19. Let S be a G-set and let $s \in S$. Show that $\mathrm{Card}(G) = \mathrm{Card}(Gs)\mathrm{Card}(G_s)$.