Math 541
Modern Algebra
A first course in Abstract Algebra
Lecturer: Arun Ram

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THE UNIVERSITY
WISCONSIN
M A D I S O N

University of Wisconsin-Madison
Mathematics Department

Homework 12: Due November 29, 2007

To grade: your grading.

Find a partner who is also in the class. Each of you should take the sample midterm below in 75 min, and then grade each others work. Grade each problem out of 20 points for a total of 100 possible points on this sample midterm. Turn in the graded sample midterms. You will be graded on how well you grade your partner's sample midterm.

> The group $G_{r,n}$ is the set of $n \times n$ matrices with
> (a) exactly one nonzero entry in each row and each column,
> (b) nonzero entries are rth roots of unity.

1. Define the following terms.
   - centralizer
   - orbit
   - G-set
   - module
   - span(S)

2. Describe the orders, centralizers and conjugacy classes of the elements of the group $G_{2,3}$.

3. Let H be a subgroup of a group G. Explain how G/H is a G-set and identify the orbits and stabilizers.

4. Give an example of a simple module for the ring $M_n(\mathbb{C})$. Don't forget to prove that your example is a simple module.

5. Let G be a group and let H be a subgroup of G.
   (a) Show that if H is index 2 in G then H is a normal subgroup of G.
   (b) Show that if H is a normal subgroup of G then H is index 2 in G.