University of Wisconsin-Madison
Mathematics Department

Math 541
Modern Algebra
A first course in Abstract Algebra
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Fall 2007

Homework 3: Due September 26, 2007

1. Define abelian group and give an example of an abelian group and a group which is not abelian.
2. Define the symmetric groups S_n.
3. Define cyclic group. What are the cyclic groups? What is the cardinality of the smallest group which is not cyclic?
4. Define the dihedral groups D_n.
5. Define the Klein 4 group.
6. Define the product of two groups. What is the smallest nontrivial example?
7. Define the general linear group, the special linear group, the orthogonal group, the special orthogonal group, the unitary group, the special unitary group, and the symplectic group.
8. Define permutation matrix.
9. Explain how the symmetric group S_n acts on a set with n elements, how the cyclic group S_n acts on an n-gon, how the dihedral group acts on an n-gon, and how the how the general linear group acts on an n -dimensional vector space.
10. Define subgroup and give some examples.
11. Define cardinality.
12. Define finite, infinite, countable and uncountable.
13. Show that $\mathrm{Card}(\mathbb{Z}_{>0}) = \mathrm{Card}(\mathbb{Z}_{\geq 0})$.
14. Show that $\mathrm{Card}(\mathbb{Z}_{\geq 0}) = \mathrm{Card}(\mathbb{Z})$.

15. Show that $\mathrm{Card}(\mathbb{Z}) = \mathrm{Card}(\mathbb{Q})$.

16. Show that $\mathrm{Card}(\mathbb{Q}) \neq \mathrm{Card}(\mathbb{R})$.

17. Show that $\mathrm{Card}(\mathbb{R}) = \mathrm{Card}(\mathbb{C})$.

18. Define homomorphism and isomorphism for groups and give some examples.

19. Define homomorphism and isomorphism for rings.

20. Define homomorphism and isomorphism for fields.

21. Let φ be a group homomorphism. Define the kernel and the image of φ and show that they are subgroups.

22. Let G and H be groups and let $\varphi : G \to H$ be a function such that if $g_1, g_2 \in G$ then $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2)$. Show that $\varphi(1) = 1$.

23. Let G and H be groups and let $\varphi : G \to H$ be a function such that if $g_1, g_2 \in G$ then $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2)$. Show that if $g \in G$ then $\varphi(g^{-1}) = \varphi(g)^{-1}$.

24. Let φ be a field homomorphism. Show that φ is injective.