

Math MAST90025 **Commutative and Multilinear Algebra Final Exam**

Lecturer: Arun Ram

2012 Semester 1 Exam ? June 2012

University of Melbourne **Mathematics** Department

This is a 2 hour exam with 200 total marks. No books, notes or calculators. Exam sheet may be retained by student.

Part I: Basic concepts

- 1. [30 marks] State and prove a theorem which makes precise the concept that equivalence relations and partitions are interchangable. Important note [15 marks out of 30]: Give precise definitions of all terms and state the theorem carefully before embarking on the proof.
- 2. [30 marks total] Prove that a function $f: X \to Y$ is an isomorphism of sets if and only if f is bijective. Important note [15 marks out of 30]: Give precise definitions of all terms and state the theorem carefully before embarking on the proof.
- 3. [40 marks total]
 - a. [5 marks] Carefully define the field of fractions of an integral domain as a set with operations.
 - b. [5 marks] Provide an alternative definition of the field of fractions of an integral domain by a universal property.
 - c. [20 marks] Prove carefully that the first definition provides a well defined object.
 - d. [10 marks] Prove that the two definitions are equivalent.

Part II: Commutative and Multilinear Algebra specifics

Write a clear proposal and description for the ideal masters level course entitled Commutative and Multilinear algebra at University of Melbourne. Be sure to include:

- 1. [5 marks] Time commitment: Contact hours and total time commitment
- 2. [5 marks] Prerequisites and recommended background knowledge
- 3. [5 marks] Subject overview
- 4. [5 marks] Objectives
- 5. [5 marks] Assessment
- 6. [5 marks] Prescribed texts and recommended texts
- 7. [10 marks] List of topics
- 8. [60 marks: 5 marks for each of 12 weeks] lecture by lecture schedule (36 lectures) with the topic of each lecture, and a list of the definitions and theorems to be covered in each lecture.