# Assignment 3

MAST90097 Algebraic Geometry
Semester II 2018
Lecturer: Arun Ram
to be turned in before 2pm on 12 October 2018

Discussions in class have indicated that students are not engaging actively with the lecture notes. This is a serious concern, particularly for preparation for the exam. The best way to engage with the lecture notes is to copy them over by hand, think them through and add annotations to fill in gaps in one's initial understanding. In view of this, there will be two options for this assignment. Please choose Option 1 or Option 2 for this assignment.

**OPTION 1**: Make a handwritten copy of the lecture notes for Weeks 1-9. For this option, 90% of the marks will be allocated for completeness, correctness of the copying, readability of the copy, and good formatting. The remaining 10% of the marks will be given for thoughtful, informative, annotations and amplifications. Please make your annotations in a different colour.

**OPTION 2**:

**1.** (30% of the marks) Carefully, precisely and accurately (i.e. in proof machine) define elliptic curve.

**2.** (70% of the marks) Elliptic curves and theta functions. For this part, use as your definition of elliptic curve the definition that you gave in part 1.

(a) Show that an elliptic curve over $\mathbb{C}$ is a Riemann surface of genus 1.

(b) Show that, as topological manifolds, all Riemann surfaces of fixed genus are isomorphic.

(c) Give (with proof) an example of two elliptic curves that are not isomorphic as complex manifolds.

(d) Show that the elliptic curves $E_\tau$ are parametrised by $\tau \in SL_2(\mathbb{Z})\backslash\mathcal{H}$, where $\mathcal{H}$ is the upper half plane.

(e) Show that $\mathcal{H} \cong SL_2(\mathbb{R})/SO_2(\mathbb{R})$.

(f) Carefully define theta functions.

(g) Carefully define elliptic functions.

(h) Determine (with proof) the minimal ample line bundle on $E_\tau$.

(i) Compute (with proof) the homogeneous coordinate ring of $E_\tau$.

(j) Show that $E_\tau$ is a projective variety by embedding it into $\mathbb{P}^2$.

(k) Classify the line bundles on $E_\tau$.

(l) Determine the structure sheaf of $E_\tau$.

(m) Carefully (with proof) explain the relationship between theta functions and the Weierstrass P-function.