Calculus 1 Lecture 23

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A. Ram

Example 3.31 Find $\frac{dy}{dx}$ if $x^2y=1$.

Solution Since $\frac{d(x^2y)}{dx}=\frac{d1}{dx}$ then

$$x^2\frac{dy}{dx}+2xy=0. \quad \text{So} \quad \frac{dy}{dx}=\frac{-2xy}{x^2}=-2\frac{y}{x}.$$

So

$$\frac{dy}{dx}=-2x^{-1}y=-2x^{-1}x^{-2}=-2x^{-3} \quad \text{if} \quad y=x^{-2}.$$

Example 3.32 Expand

$$\frac{d}{dx}(\sin y), \quad \frac{d}{dx}(y^3), \quad \frac{d}{dx}(xe^y) \text{ and } \frac{d}{dx}(x^2\log y).$$

Solution By the chain rule,

$$\frac{d(\sin y)}{dx}=\cos y\frac{dy}{dx},$$

$$\frac{d}{dx}(y^3)=3y^2\frac{dy}{dx},$$

$$\frac{d}{dx}(xe^y)=xe^y\frac{dy}{dx}+e^y, \quad \text{and}$$

$$\frac{d}{dx}(x^2\log(y))=x^2\frac{1}{y}\frac{dy}{dx}+2x\log(y).$$

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Example 3.33 Let $x^2 - xy + y^4 = 16$.

Solution: Take the derivative of both sides to get

$$2x - x\frac{dy}{dx} - y + 4y^3\frac{dy}{dx} = 0.$$

If $x=0$ then $y^4=16$ and $y=\pm 2$

If $y=0$ then $x^2=16$ and $x=\pm 4$.

Find $\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=2}}$ by $2\cdot 0 - 0 - 2 + 4\cdot 2^3 \left.\frac{dy}{dx}\right|_{\substack{x=0\\y=2}} = 0$.

So $\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=2}} = \frac{2}{4\cdot 2^3} = \frac{1}{2^4} = \frac{1}{16}$

Find $\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=-2}}$ by $2\cdot 0 - 0 - (-2) + 4(-2)^3 \left.\frac{dy}{dx}\right|_{\substack{x=0\\y=2}} = 0$

So $\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=2}} = \frac{-2}{4(-2)^3} = \frac{1}{2^4} = \frac{1}{16}$.

If $x=y$ then $x^2 - x^2 + x^4 = 16$ and $x=\pm 2$.

Find $\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}}$ by $2\cdot 2 - 2\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}} - 2 + 4\cdot 2^3 \left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}} = 0$

So $\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}} = \frac{-2}{14} = \frac{-1}{7}$.

Find $\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=-2}}$ by $2(-2)-(-2)\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=-2}}-(-2)+4(-2)^3\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=-2}}=0.$

So $\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=-2}}=\frac{-1}{7}.$

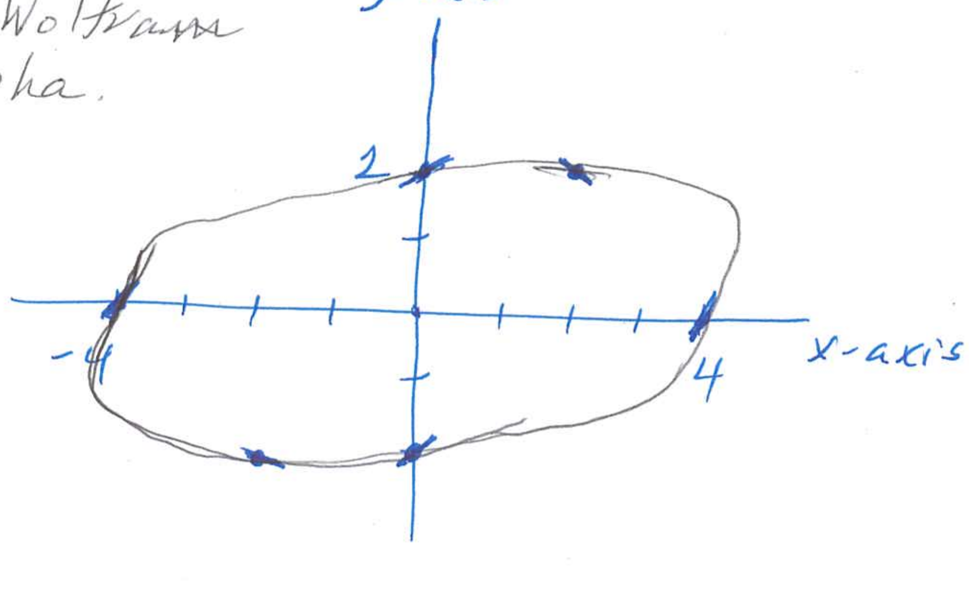
Find $\left.\frac{dy}{dx}\right|_{\substack{x=4\\y=0}}$ by $2\cdot4-4\left.\frac{dy}{dx}\right|_{\substack{x=4\\y=0}}-0+4\cdot0^3\cdot\left.\frac{dy}{dx}\right|_{\substack{x=4\\y=0}}=0$

So $\left.\frac{dy}{dx}\right|_{\substack{x=4\\y=0}}=2$

Find $\left.\frac{dy}{dx}\right|_{\substack{x=-4\\y=0}}$ by $2(-4)-(-4)\cdot\left.\frac{dy}{dx}\right|_{\substack{x=-4\\y=0}}-0+4\cdot0^3\left.\frac{dy}{dx}\right|_{\substack{x=-4\\y=0}}=0$

So $\left.\frac{dy}{dx}\right|_{\substack{x=-4\\y=0}}=2.$

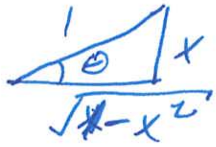
Using Wolfram alpha.



Example 3.38 Compute $\cos(\arcsin x)$.

Solution Let $\theta = \arcsin x$.

Then $\sin\theta = x = \frac{x}{1}$



So $\cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$.

So $\cos(\arcsin x) = \sqrt{1-x^2}$.

Example Compute $\frac{d}{dx}(\arcsin x)$.

Solution Let $\theta = \arcsin x$. So $\sin\theta = x$.

So $\frac{d}{dx}(\sin\theta) = \frac{dx}{dx}$. So $\cos\theta \frac{d\theta}{dx} = 1$.

So $\frac{d\theta}{dx} = \frac{1}{\cos\theta} = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}}$.

So $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.

Example Compute $\frac{d}{dx}(\arctan x)$

Solution Let $\theta = \arctan x$. So $\tan\theta = x$.

So $\frac{d}{dx}(\tan\theta) = \frac{dx}{dx}$. So $\sec^2\theta \frac{d\theta}{dx} = 1$.

So $\frac{d}{dx}(\arctan x) = \frac{d\theta}{dx} = \frac{1}{\sec^2\theta} = \frac{1}{\tan^2\theta + 1} = \frac{1}{x^2+1}$. //.