Calculus I Lecture 25

Example 3.42 Find $\vec{r}\,'(t)$.

(a) $\vec{r}(t) = (t^2+1)\hat{\imath} + (\frac{1}{3}t^3+1)\hat{\jmath}$

So $\vec{r}\,'(t) = \frac{d\vec{r}}{dt} = 2t\hat{\imath} + (\frac{1}{3}3t^2+0)\hat{\jmath}$

$= 2t\hat{\imath} + t^2\hat{\jmath}$.

(b) $\vec{r}(t) = \hat{\imath} + 3t^3\hat{\jmath}$

So $\vec{r}\,'(t) = \frac{d\vec{r}}{dt} = \frac{d1}{dt}\hat{\imath} + 3\cdot 3t^2\hat{\jmath} = 0\hat{\imath} + 9t^2\hat{\jmath} = 9t^2\hat{\jmath}$.

(c) $\vec{r}(t) = 2t\hat{\imath} - (3t^3+1)\hat{\jmath}$

So $\vec{r}\,'(t) = \frac{d\vec{r}}{dt} = 2\hat{\imath} - (3\cdot 3t^2+0)\hat{\jmath} = 2\hat{\imath} - 9t^2\hat{\jmath}$

(d) $\vec{r}(t) = 2\cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$.

So $\vec{r}\,'(t) = -2\sin t\,\hat{\imath} + \cos t\,\hat{\jmath}$.

Example 3.43 Let $\vec{r}(t) = 2\cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$.

Then $\vec{r}\,'(t) = -2\sin t\,\hat{\imath} + \cos t\,\hat{\jmath}$.

So $\vec{r}\,'(0) = -2\sin 0\,\hat{\imath} + \cos 0\,\hat{\jmath} = 0\hat{\imath} + \hat{\jmath} = \hat{\jmath}$.

$\vec{r}\,'(\frac{\pi}{2}) = -2\sin\frac{\pi}{2}\hat{\imath} + \cos\frac{\pi}{2}\hat{\jmath} = -2\cdot 1\hat{\imath} + 0\hat{\jmath} = -2\hat{\imath}$

$\vec{r}\,'(\pi) = -2\sin\pi\,\hat{\imath} + \cos\pi\,\hat{\jmath} = -2\cdot 0\hat{\imath} - \hat{\jmath} = -\hat{\jmath}$

$\vec{r}\,'(\frac{3\pi}{2}) = -2\sin\frac{3\pi}{2}\hat{\imath} + \cos\frac{3\pi}{2}\hat{\jmath} = (-2)(-1)\hat{\imath} + 0\hat{\jmath} = 2\hat{\imath}$.

Example 3.44 Find the speed at time t.

(a) $\vec{r}(t) = (t^2+1)\hat{i} + (\frac{1}{3}t^3+1)\hat{j}$.

So $\vec{r}\,'(t) = 2t\hat{i} + t^2\hat{j}$

$$\text{speed} = |\vec{r}\,'(t)| = \sqrt{(2t)^2+(t^2)^2} = \sqrt{4t^2+t^4}$$
$$= t\sqrt{4+t^2}$$

(b) $\vec{r}(t) = 2t\hat{i} - (3t^3+1)\hat{j}$.

So $\vec{r}\,'(t) = 2\hat{i} - 9t^2\hat{j}$ and

$$\text{speed} = |\vec{r}\,'(t)| = \sqrt{2^2+(-9t^2)^2} = \sqrt{4+81t^4}.$$

(c) $\vec{r}(t) = 2\cos(t)\hat{i} + \sin t\,\hat{j}$

So $\vec{r}\,'(t) = -2\sin t\,\hat{i} + \cos t\,\hat{j}$ and

$$\text{speed} = |\vec{r}\,'(t)| = \sqrt{4\sin^2 t + \cos^2 t} = \sqrt{1+3\sin^2 t}.$$

Example 3.46 $\vec{r}(t) = t^3\hat{i} + t^5\hat{j}$.

So $\vec{r}\,'(t) = 3t^2\hat{i} + 5t^4\hat{j}$ and

$$\text{speed} = |\vec{r}\,'(t)| = \sqrt{9t^4+25t^8} = t^2\sqrt{9+25t^4}$$

So speed is 0 when $t=0$

~~But speed is always ≥ 0 so~~ And $\vec{r}(t)$ ~~does not~~ changes ~~direction~~ from negative to positive as ~~t~~ t goes from negative to positive.

Example 3.48 Find the acceleration.

(a) $\vec{r}(t) = (t^2+1)\hat{i} + (\frac{1}{3}t^3+1)\hat{j}$

So $\vec{r}\,'(t) = 2t\hat{i} + t^2\hat{j}$ and

acceleration $= \vec{r}\,''(t) = 2\hat{i} + 2t\hat{j}$.

(b) $\vec{r}(t) = 2t\hat{i} - (3t^3+1)\hat{j}$ and

~~acceleration $= \vec{r}\,''(t) = 2$~~

$\vec{r}\,'(t) = 2\hat{i} - 9t^2\hat{j}$ and

acceleration $= \vec{r}\,''(t) = 0\hat{i} - 18t\hat{j} = -18t\hat{j}$.

(c) $\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}$

So $\vec{r}\,'(t) = (1-\cos t)\hat{i} + (\sin t)\hat{j}$ and

acceleration $= \vec{r}\,''(t) = \sin t\,\hat{i} + \cos t\,\hat{j}$.

Example 3.50 Find t for which the speed is increasing.

(a) $\vec{r}(t) = (t^2+1)\hat{i} + (\frac{1}{3}t^3+1)\hat{j}$.

So $\vec{r}\,'(t) = 2t\hat{i} + t^2\hat{j}$ and

s = speed $= \sqrt{4t^2+t^4} = |t|\sqrt{4+t^2}$.

$\frac{ds}{dt} = (4+t^2)^{\frac{1}{2}} + t\,\frac{1}{2}(4+t^2)^{-\frac{1}{2}} \cdot 2t =$ ~~$\frac{\ }{(4+t^2)^{\frac{1}{2}}}$~~ $*$

$= (4+t^2)^{\frac{1}{2}} + \frac{t^2}{(4+t^2)^{\frac{1}{2}}} > 0$ if $t \in \mathbb{R}_{\geq 0}$.

So the speed is increasing for $t \in \mathbb{R}_{\geq 0}$.

(b) $\vec{r}(t) = (t - \sin t)\hat{i} + (1-\cos t)\hat{j}$

So $\vec{r}'(t) = (1-\cos t)\hat{i} + \sin t\,\hat{j}$ and

$$s = \text{speed} = \sqrt{(1-\cos t)^2 + \sin^2 t}$$
$$= \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}$$
$$= \sqrt{2 - 2\cos t} = \sqrt{2}\,(1-\cos t)^{\frac{1}{2}}.$$

So $\frac{ds}{dt} = \sqrt{2}\,\frac{1}{2}\,(1-\cos t)^{-\frac{1}{2}} \sin t = \frac{1}{\sqrt{2}(1-\cos t)^{\frac{1}{2}}} \sin t$

which is positive when $\sin t > 0$.

So $\frac{ds}{dt}$ is increasing if $t \in \mathbb{R}_{(0,\pi)} \cup \mathbb{R}_{(2\pi, 3\pi)} \cup \ldots$