Example 4.39 Solve $\frac{dy}{dx}=y$.

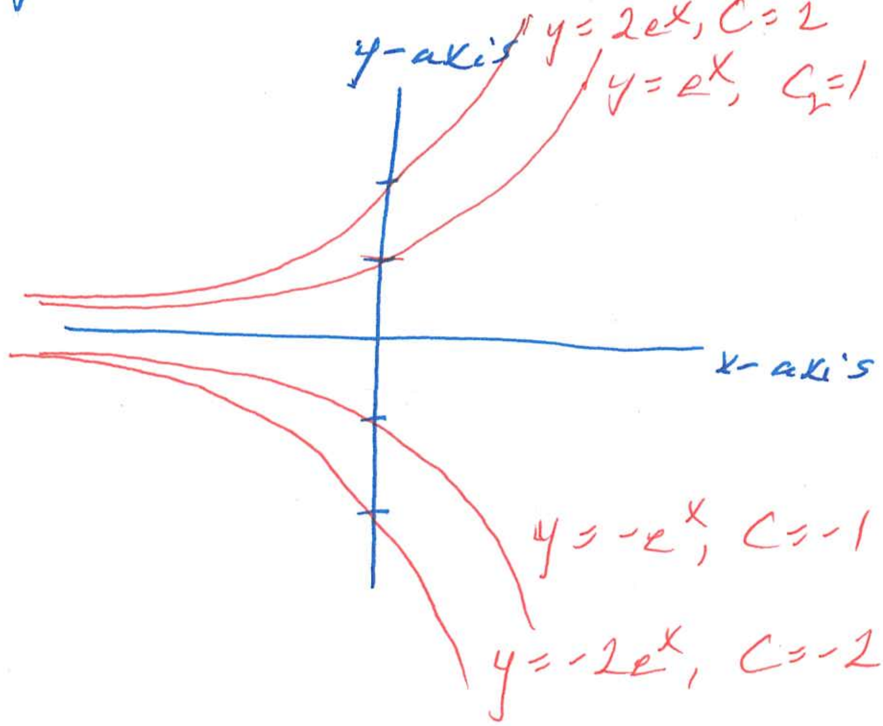
Solution: $\frac{dy}{dx}=y$. So $\frac{1}{y}\frac{dy}{dx}=1$.

So $\int \frac{1}{y}\frac{dy}{dx}\,dx=\int dx$. So $\log y = x+c_1$,

where c_1 is a constant. So

$e^{\log y}=e^{x+c_1}=e^x e^{c_1}$.

So $y=e^x C_2$, where C_2 is a constant.



The solution $y=-e^x$ has $y=-1$ when $x=0$.

Example 4.41 Solve $\frac{dy}{dx} = -6xy^2$.

Solution $\frac{dy}{dx} = -6xy^2$. So $\frac{1}{y^2}\frac{dy}{dx} = -6x$.

So $\int \frac{1}{y^2}\frac{dy}{dx}\,dx = \int -6x\,dx$. So $\int y^{-2}\,dy = \int -6x\,dx$

So $\frac{1}{-1}y^{-1} = -3x^2 + c$, where c is a constant.

So $\frac{1}{y} = 3x^2 - c$. So $y = \frac{1}{3x^2 - c}$.

Example 4.43 Solve $\frac{dy}{dx} = \frac{1}{6y^5 - 2y + 1}$.

Solution Since $\frac{dy}{dx} = \frac{1}{6y^5 - 2y + 1}$ then

$(6y^5 - 2y + 1)\frac{dy}{dx} = 1$. So $\int (6y^5 - 2y + 1)\frac{dy}{dx}\,dx = \int 1\,dx$.

So $\int (6y^5 - 2y + 1)\,dy = \int dx$.

So $y^6 - y^2 + y = x + c$, where c is a constant.

This is an "implicit" expression for y.

<u>Example 4.44</u> "Separate" $\frac{dy}{dx} = y^2 \sin x + y^2 - 4\sin x - 4$.

<u>Solution</u> $\frac{dy}{dx} = y^2(\sin x + 1) - 4(\sin x + 1)$

So $\frac{dy}{dx} = (y^2-4)(\sin x + 1)$

So $\frac{1}{y^2-4}\frac{dy}{dx} = \sin x + 1.$

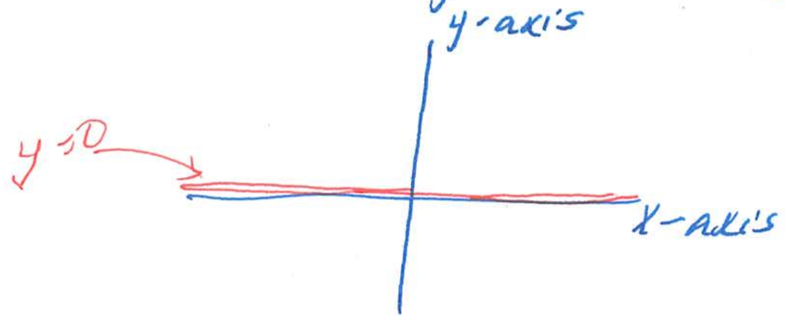
So $\left(\frac{\frac{1}{4}}{y-2} + \frac{-\frac{1}{4}}{y+2}\right)\frac{dy}{dx} = \sin x + 1$

So $\int\left(\frac{\frac{1}{4}}{y-2} + \frac{-\frac{1}{4}}{y+2}\right)\frac{dy}{dx}\,dx = \int(\sin x + 1)\,dx.$

So $\frac{1}{4}\log(y-2) - \frac{1}{4}\log(y+2) = -\cos x + x + c,$
where c is a constant.

<u>Example 4.45</u> Solve $\frac{dy}{dx} = -6xy^2$ if $y=0$ when $x=0$.

<u>Solution</u> $y=0$ is a solution to $\frac{dy}{dx} = -6xy^2$ which satisfies $y=0$ when $x=0$.



Example 4.46 Find constant solutions of

(a) $\frac{dy}{dx} = y - \frac{y^2}{4}$ (b) $\frac{dy}{dx} = e^y \sin x$

(c) $\frac{dy}{dx} = \cos^2 y \, e^x$ (d) $\frac{dy}{dx} = y$ (e) $\frac{dy}{dx} = y^{2/3}$.

Solution If y is constant then $\frac{dy}{dx} = 0$.

(a) $0 = y - \frac{y^2}{4} = (1 - \frac{y}{4})y$ gives $y = 0$ or $y = 4$.

(b) $0 = e^y \sin x$ gives $e^y = 0$ or $\sin x = 0$
This does not yield solutions that exists on intervals.

(c) $0 = \cos^2 y \cdot e^x$ gives $\cos^2 y = 0$ and
$y = \frac{\pi}{2}$ or $y = \frac{3\pi}{2}$ or $y = \frac{5\pi}{2}$ or ....

(d) $0 = y$ gives $y = 0$

(e) $0 = y^{2/3}$ gives $y = 0$.

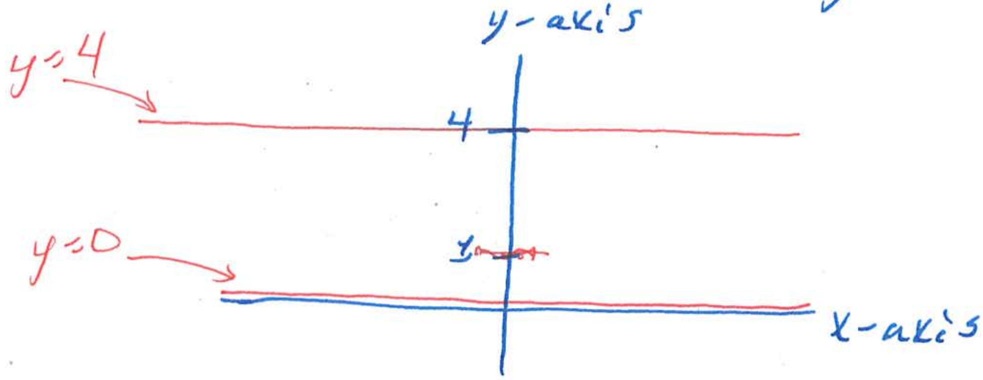
Example 4.51 Show that the solution of

$\frac{dy}{dx} = y - \frac{y^2}{4}$ which has $y(0) = 1$

also has $0 < y(x) < 4$ for $x \in \mathbb{R}$.

Solution Since $y - \frac{y^2}{4}$ is continuous (polynomials) and differentiable and $y=0$ and $y=4$ are constant solutions of $\frac{dy}{dx} = y - \frac{y^2}{4}$ then the solution with

$0 < y(0) = 1 < 4$ will always have $0 < y(x) < 4$.



Example 4.52 Solve $\frac{dy}{dx} = y^{1/3}$.

Solution If $\frac{dy}{dx} = 0$ then $y^{1/3} = 0$ and $y = 0$.

If $\frac{dy}{dx} = y^{1/3}$ then (assuming $y^{1/3} \neq 0$)

$y^{-1/3} \frac{dy}{dx} = 1$. So $\int y^{-1/3} \frac{dy}{dx}\, dx = \int 1\, dx$.

So $\frac{3}{2} y^{2/3} = x + c_1$, where c_1 is a constant.

So $y^{2/3} = \frac{2}{3}x + c_2$, where c_2 is a constant.

So $y = \left(\frac{2}{3}x + c_2\right)^{3/2}$.

Draw pictures/graphs of these solutions.

Calc. I Lect 33 17.10.2019

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(6)