MAST10005 Slide

A. Ram 23.07.2019

Complex numbers

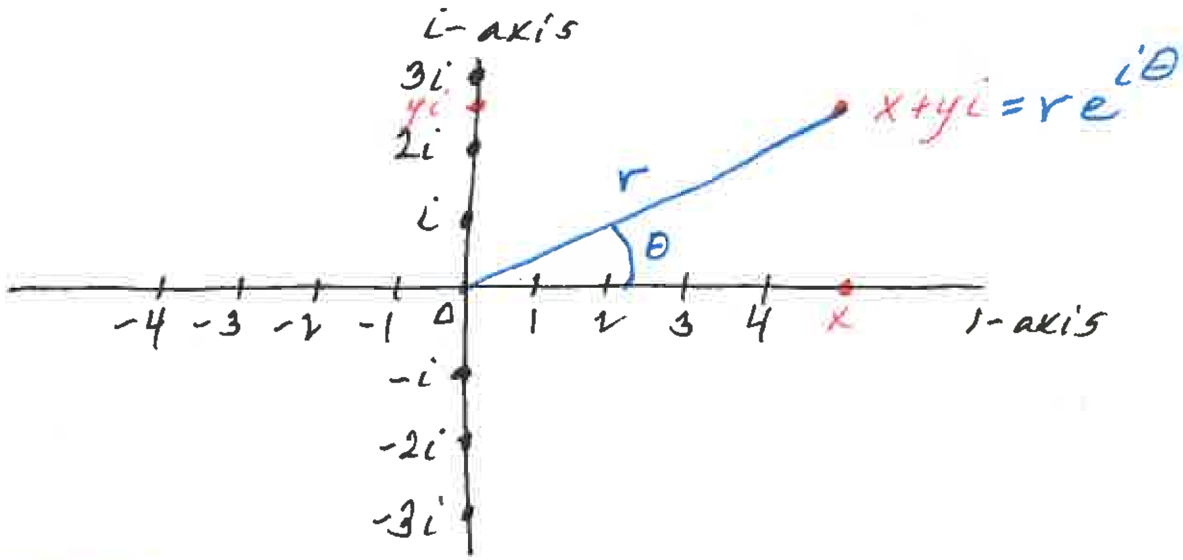
$\mathbb{C} = \{a+bi \mid a,b \in \mathbb{R}\}$ with $i^2 = -1$.

$(a+bi)+(c+di) = (a+c)+(b+d)i$ (addition)

$c(a+bi) = ca+cbi$ (scalar multiplication)

$(a+bi)(c+di) = ac+adi+bci+bdi^2$

$= (ac-bd)+(ad+bc)i$ (multiplication)



Graphing of real and complex numbers.

$\overline{a+bi} = a-bi$ (conjugation)

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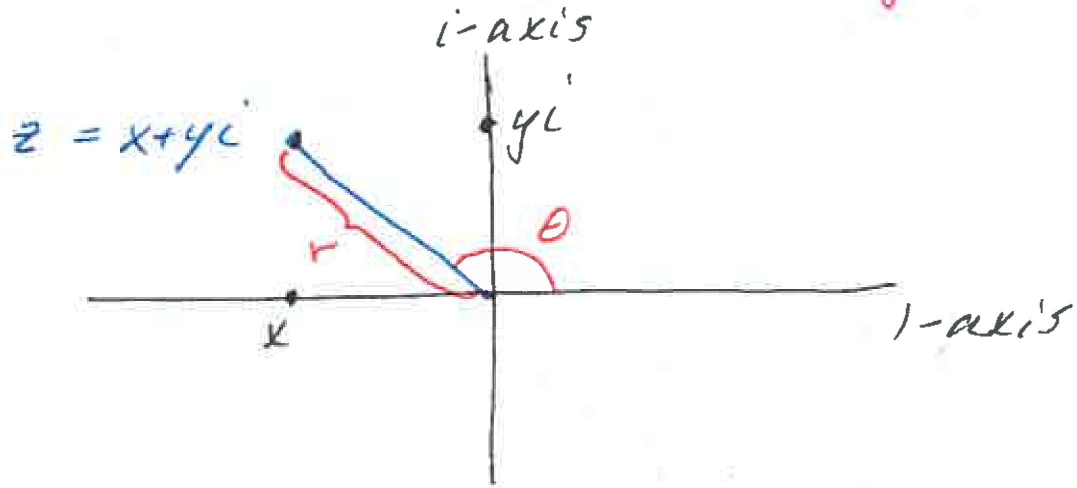
Complex numbers

$z = x + yi = re^{i\theta}$ with $x \in \mathbb{R}$, $y \in \mathbb{R}$

$r \in \mathbb{R}_{>0}$, $\theta \in \mathbb{R}_{[0, 2\pi)}$

$\mathrm{Re}(z)$ $\mathrm{Im}(z)$ $|z|$ $\arg(z)$



$|z|$ is the modulus of z.

$x = r\cos\theta$, $y = r\sin\theta$

$re^{i\theta} = r\cos\theta + r\sin\theta\, i$, since

$r = \sqrt{x^2 + y^2}$, $\theta = \arctan(y/x)$

Lecture 4

Calculus I: Complex Numbers Examples A. Ram ①

Example 1.27 Find solutions of $x^2+4=0$.

Solution: If $x^2+4=(x+2i)(x-2i)=0$

then $x=-2i$ or $x=2i$.

Example 1.28 Find solutions of $x^2-2x+5=0$.

Solution: $0=x^2-2x+5 = x^2-2x+1+4 = (x-1)^2+4$

$$= (x-1+2i)(x-1-2i)$$

So $x = 1-2i$ or $x=1+2i$.

Example 1.30 (a) Find $\sqrt{-25}$

(b) Simplify i^7.

Solution: (a) $\sqrt{-25} = \sqrt{-1}\sqrt{25} = \pm i5 = \pm 5i$.

If $x^2=-25$ then $x^2+25=0$ so $(x+5i)(x-5i)=0$.

So $x=5i$ or $x=-5i$.

(b) $i^7 = i^2 \cdot i^2 \cdot i^2 \cdot i = (-1)(-1)(-1)i = -i$.

Example 1.32 Let $z=2-3i$. Find $Re(z)$, $Im(z)$ and $Re(z)-Im(z)$.

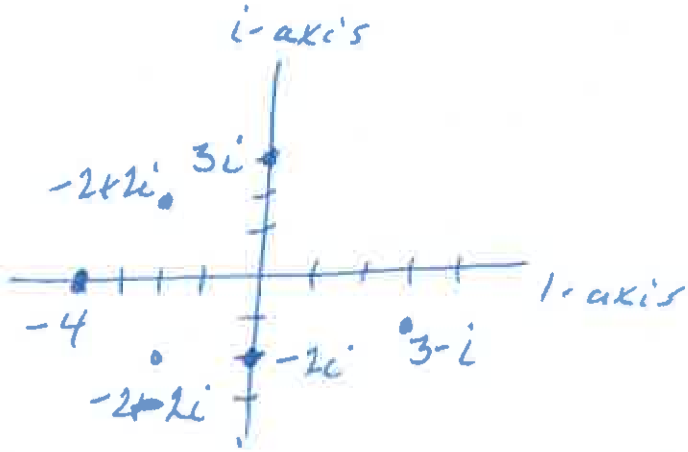
Solution: $Re(z) = Re(2-3i) = 2$.

$Im(z) = Im(2-3i) = -3$.

$Re(z)-Im(z) = 2-(-3) = 2+3 = 5$.

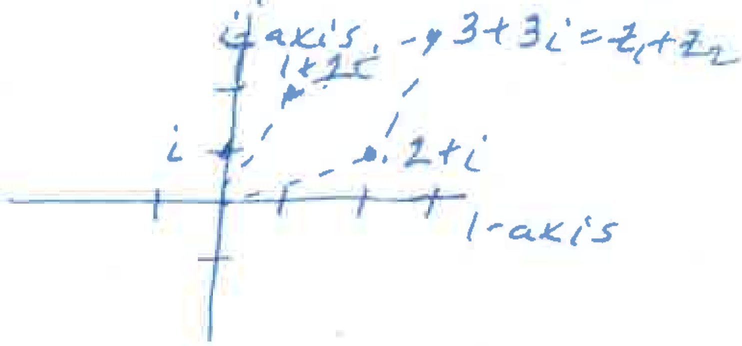
Example 1.33 Graph $3-i$, $-2+2i$, -4 and $3i$.

Solution:



Example 1.34 Graph $z_1 = 1+2i$, $z_2 = 2+i$ and z_1+z_2

Solution:



Example 1.36 Compute $\overline{-3+7i}$, $\overline{2-5i}$, $\overline{3i}$, $\overline{4}$.

Solution: $\overline{-3+7i} = -3-7i$,

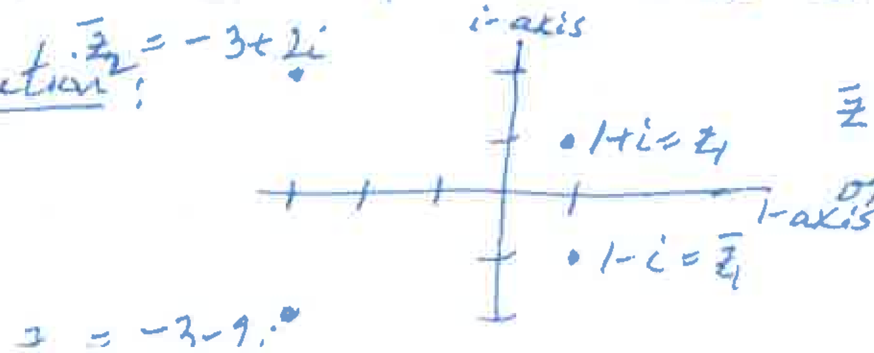
$\overline{2-5i} = 2+5i$,

$3i = -3i$,

$4 = 4$.

Example 1.37 Graph $z_1 = 1+i$, $z_2 = -3-2i$, $\bar{z}_1$ and $\bar{z}_2$.

Solution:



$\bar{z}$ is the reflection of z in the 1-axis.

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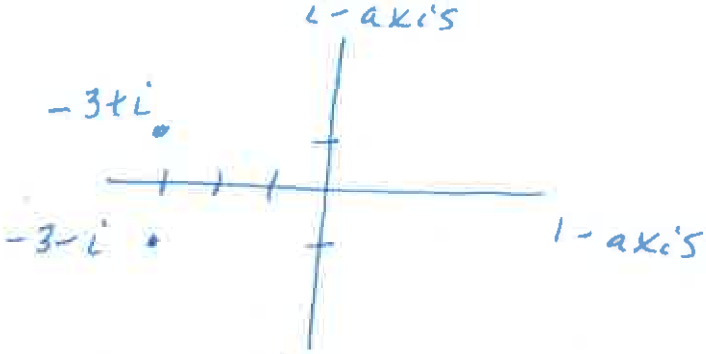
Example 1.38 Graph solutions of

$$z^2 - 6z + 10 = 0.$$

Solution $0 = z^2 - 6z + 10 = z^2 - 6z + 9 + 1$

$$= (z-3)^2 + 1 = (z-3+i)(z-3-i)$$

So $z = -3+i$ or $z = -3-i = \overline{-3+i}$.



Example 1.40 Let $z = x+iy$ and $w = a+ib$.

(a) Prove that $z + \bar{z} = 2x = 2\,\mathrm{Re}(z)$

(b) Prove that $z - \bar{z} = 2yi = 2\,\mathrm{Im}(z)\,i$

(c) Prove that $z\bar{z} = x^2 + y^2$

Solution: (a) $z + \bar{z} = (x+iy) + (x-iy) = 2x = 2\,\mathrm{Re}(z)$

(b) $z - \bar{z} = (x+iy) - (x-iy) = 2iy = 2yi = 2\,\mathrm{Im}(z)\,i$.

(c) $z\bar{z} = (x+iy)(x-iy) = x^2 - xiy + iyx - i^2y^2$

$$= x^2 - (-1)y^2 = x^2 + y^2$$